

2022 年河南省普通高中 招生考试数学预测卷(四)

快速对答案

1. C 2. D 3. A 4. D 5. A 6. C 7. A 8. C

9. A 10. A 11. $\sqrt{2}$ 12. 25° 13. $3 < m < 5$

14. $\frac{15}{4}\pi - \frac{9\sqrt{3}}{2}$ 15. $\frac{1}{3}$ 16. $-\frac{\sqrt{2}}{2}$

17. (1) 60 (2) 见解析 (3) 108 (4) 200 名

18. (1) 92.5 cm (2) 29.3 cm

19. (1) $(-4, 1)$ (2) $1 < m < 9$

20. (1) 30° (2) $\sqrt{3}$ (3) $\frac{36}{5}$

21. (1) 每把椅子 40 元, 每张桌子 320 元 (2) 见解析 (3) 见解析

22. (1) $\sqrt{3}$ (2) 90° (3) $2\sqrt{3}$

23. (1) $y = -\frac{3}{4}x^2 + \frac{9}{4}x + 3$ (2) $0 \leq x \leq 3$

(3) $\left(\frac{3}{2}, 2 + \frac{\sqrt{6}}{3}\right)$ 或 $\left(\frac{3}{2}, 2 - \frac{\sqrt{6}}{3}\right)$

重点题目解析

9. A 【解析】由题可得 $BN = x$. 当 $0 \leq x \leq 1$ 时, 点 M 在

BC 边上, $BM = 3x$, $AN = 3 - x$, 则 $S_{\triangle ANM} = \frac{1}{2}AN \cdot BM$,

$\therefore y = \frac{1}{2} \cdot (3 - x) \cdot 3x = -\frac{3}{2}x^2 + \frac{9}{2}x$, 故 C 选项

错误; 当 $1 < x < 2$ 时, 点 M 在 CD 边上, 则 $S_{\triangle ANM} =$

$\frac{1}{2}AN \cdot BC$, $\therefore y = \frac{1}{2} \cdot (3 - x) \cdot 3 = -\frac{3}{2}x + \frac{9}{2}$, 故

D 选项错误; 当 $2 \leq x \leq 3$ 时, 点 M 在 AD 边上,

$AM = 9 - 3x$, $\therefore S_{\triangle ANM} = \frac{1}{2}AM \cdot AN$, $\therefore y = \frac{1}{2} \cdot$

$(9 - 3x) \cdot (3 - x) = \frac{3}{2}(x - 3)^2$, 故 B 选项错误.

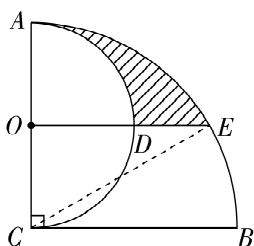
故选 A.

10. A 【解析】 \because 正方形 $ABCD$, 顶点 $A(1, 3)$, $B(1, 1)$, $C(3, 1)$, \therefore 对角线交点 M 的坐标为 $(2, 2)$.

根据题意得第 1 次变换后的点 M 的对应点的坐标为 $(2 - 1, -2)$, 即 $(1, -2)$; 第 2 次变换后的点 M 的对应点的坐标为 $(2 - 2, 2)$, 即 $(0, 2)$; 第 3 次变换后的点 M 的对应点的坐标为 $(2 - 3, -2)$, 即 $(-1, -2)$; 第 n 次变换后的点 M 的对

应点的坐标:当 n 为奇数时,为 $(2-n, -2)$; 当 n 为偶数时,为 $(2-n, 2)$, \therefore 连续经过 2 022 次变换后,正方形 $ABCD$ 的对角线交点 M 的坐标变为 $(-2\ 020, 2)$. 故选 A.

14. $\frac{15}{4}\pi - \frac{9\sqrt{3}}{2}$ 【解析】如图,连接 CE .



$\because \angle ACB = 90^\circ, AC = BC = 6$, 以 AC 为直径作半圆, 圆心为点 O , $\therefore OA = OC = OD = 3, BC = CE = CA = 6$. 又 $\because OE \parallel BC$, $\therefore \angle AOE = \angle COE = \angle ACB = 90^\circ$, \therefore 在 $\text{Rt} \triangle OEC$ 中, $OC = \frac{1}{2} CE$, $\therefore \angle OEC = 30^\circ, OE = 3\sqrt{3}$, $\therefore \angle ECB = \angle OEC = 30^\circ$, $\therefore S_{\text{阴影}} = S_{\text{扇形}ACB} - S_{\text{扇形}AOD} - S_{\text{扇形}ECB} - S_{\triangle OCE} = \frac{90\pi \times 6^2}{360} - \frac{90 \cdot \pi \times 3^2}{360} - \frac{30 \cdot \pi \times 6^2}{360} - \frac{1}{2} \times 3 \times 3\sqrt{3} = \frac{15}{4}\pi - \frac{9\sqrt{3}}{2}$. 故答案为 $\frac{15}{4}\pi - \frac{9\sqrt{3}}{2}$.

15. $\frac{1}{3}$ 【解析】连接 AE . 由翻折可得 $C'E = CE$. \because 点

E 是 CD 边的中点, $\therefore DE = CE$, $\therefore C'E = DE$.

$\because AE = AE, AD = AC', C'E = DE$, $\therefore \triangle ADE \cong \triangle AC'E$ (SSS),

$\therefore \angle ADE = \angle AC'E = 90^\circ$.

$\because \angle C = \angle FC'E = 90^\circ$,

$\therefore \angle AC'E + \angle FC'E = 180^\circ$,

\therefore 点 A, C', F 共线. 设 $CF = x$, 则 $BF = 3 - x, AF = 3 + x$.

在 $\text{Rt} \triangle ABF$ 中, $2^2 + (3 - x)^2 = (3 + x)^2$, 解得 $x = \frac{1}{3}$. 故 CF 的长为 $\frac{1}{3}$.

16. 【解】原式 $= \frac{x^2 - 2x}{x^2 - 1} \div \left(\frac{2x - 1}{x + 1} - \frac{x^2 - 1}{x + 1} \right) = \frac{x^2 - 2x}{x^2 - 1} \times$

$$\frac{x + 1}{2x - x^2} = -\frac{1}{x - 1}. \quad (6 \text{ 分})$$

当 $x = \sqrt{2} + 1$ 时, 原式 $= -\frac{1}{\sqrt{2} + 1 - 1} = -\frac{\sqrt{2}}{2}$.

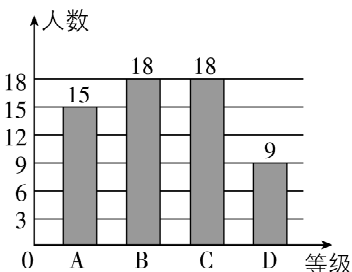
(10 分)

17. (1) 60

(2 分)

【解析】本次随机调查的学生人数为 $15 \div 25\% = 60$.
故答案为 60.

(2) 【解】 $60 - 15 - 18 - 9 = 18$ (人), 补全条形统计图如图所示:



(5 分)

(3) 108

(7 分)

【解析】在扇形统计图中, “C” 所在扇形的圆心角为 $360^\circ \times \frac{18}{60} = 108^\circ$, 故答案为 108.

(4) 【解】 $800 \times \frac{15}{60} = 200$ (名).

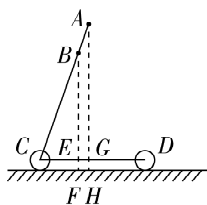
(9 分)

18. 【解】(1) 如图, 过点 B 作 $BE \perp CD$ 于点 E , 延长 BE 交地面于点 F .

$\therefore \sin \angle BCE = \frac{BE}{BC}, \therefore BE = BC \times \sin 70^\circ \approx 92 \times 0.94 = 86.48$.

$\therefore EF = 6, \therefore BF = BE + EF \approx 92.5$ cm. \therefore 固定车杆 BC 的上端 B 离地面的高度约为 92.5 cm.

(4 分)



(2) 如图, 过点 A 作 $AG \perp CD$ 于点 G , 延长 AG 交地面于点 H .

$\therefore AH = 120, GH = 6, \therefore AG = 114$.

$\therefore \sin \angle ACG = \frac{AG}{AC},$

$\therefore AC = \frac{AG}{\sin 70^\circ} \approx \frac{114}{0.94} \approx 121.3,$ (7 分)

$\therefore AB = AC - BC = 121.3 - 92 = 29.3$ (cm).

\therefore 此时伸缩杆 AB 的长度约为 29.3 cm. (9 分)

19. 【解】(1) 把 $A(-1, b)$ 代入 $y = -\frac{4}{x}$ 得 $b = 4$,

所以 A 点坐标为 $(-1, 4)$.

把 $A(-1, 4)$ 代入 $y = kx + 5$ 得 $-k + 5 = 4$, 解得

$$k=1,$$

所以一次函数解析式为 $y=x+5$.

$$\text{联立} \begin{cases} y=x+5, \\ y=\frac{-4}{x}, \end{cases}$$

$$\text{解得} \begin{cases} x=-1, \\ y=4 \end{cases}, \text{或} \begin{cases} x=-4, \\ y=1, \end{cases}$$

则 B 点坐标为 $(-4, 1)$. (4 分)

(2) 将直线 AB 向下平移 m ($m > 0$) 个单位长度得直线解析式为 $y=x+5-m$,

$$\text{联立} \begin{cases} y=x+5-m, \\ y=\frac{-4}{x}, \end{cases}$$

整理得 $x^2 - (m-5)x + 4 = 0$. 因为两图象没有公共点,

所以 $[-(m-5)]^2 - 4 \times 1 \times 4 < 0$, 解得 $1 < m < 9$,

即 m 的取值范围为 $1 < m < 9$. (9 分)

20. 【解】(1) 连接 OC .

$\because \angle A = \angle B, \therefore OA = OB. \because CA = CB,$

$\therefore OC \perp AB. \because OE = OC, \therefore \angle OEC = \angle OCE = 30^\circ,$

$\therefore \angle BOC = \angle OEC + \angle OCE = 60^\circ. \because OC = OD,$

$\therefore \triangle OCD$ 为等边三角形, $\therefore \angle OCD = 60^\circ.$

$\because OC \perp AB, \therefore \angle OCB = 90^\circ, \therefore \angle DCB = 30^\circ.$

(3 分)

(2) $\because \angle BOC = 60^\circ, OC = OD, \therefore \triangle OCD$ 为等边

三角形, $\therefore S_{\triangle OCD} = \frac{1}{2} \times 2 \times \sqrt{3} = \sqrt{3}. \because \angle BOC =$

$60^\circ, \angle OCB = 90^\circ, OC = 2, \therefore BC = 2\sqrt{3},$

$\therefore S_{\triangle OCB} = \frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3}, \therefore S_{\triangle BCD} = S_{\triangle OCB} -$

$S_{\triangle OCD} = \sqrt{3}.$ (6 分)

(3) $\because ED$ 是直径, $\therefore \angle ECD = 90^\circ, \therefore \angle E + \angle EDC = 90^\circ.$

又 \because 由 (1) 知 $\angle BCD + \angle OCD = 90^\circ, \angle OCD = \angle ODC, \therefore \angle BCD = \angle E.$

又 $\because \angle CBD = \angle EBC, \therefore \triangle BCD \sim \triangle BEC,$

$$\therefore \frac{BC}{BE} = \frac{BD}{BC} = \frac{CD}{EC} = \frac{2}{3}.$$

设 $BD = x$, 则 $BC = \frac{3}{2}x, BE = x + 4.$

又 $\because BC^2 = BD \cdot BE, \therefore \left(\frac{3}{2}x\right)^2 = x(x + 4),$ 解得

$$x_1 = 0, x_2 = \frac{16}{5}.$$

$$\begin{aligned} \because BD = x > 0, \therefore BD = \frac{16}{5}, \therefore BE = BD + DE = \frac{16}{5} + 4 = \frac{36}{5}. \end{aligned} \quad (9 \text{ 分})$$

21. 【解】(1) 设该学校购买的每把椅子 x 元, 每张桌子 y 元.

$$\begin{aligned} \text{由题意得} \quad & \begin{cases} y = 8x, \\ 1200x + 300y = 144000, \end{cases} \quad \text{解} \\ \text{得} \quad & \begin{cases} x = 40, \\ y = 320. \end{cases} \end{aligned}$$

答: 学校购买的每把椅子 40 元, 每张桌子 320 元.

(3 分)

(2) 设租用 A 型卡车 n 辆, 则租用 B 型卡车 $(20 - n)$ 辆.

$$\text{由题意得} \quad \begin{cases} 70n + 45(20 - n) \geq 1200, \\ 10n + 30(20 - n) \geq 300, \end{cases} \quad \text{解得 } 12 \leq$$

$n \leq 15$. 由题意可知 n 为正整数, 所以 n 只能取 12, 13, 14, 15, 故有四种租车方案可一次性将这批餐桌椅运回来, 可这样安排:

方案一: A 型卡车 12 辆, B 型卡车 8 辆; 方案二: A 型卡车 13 辆, B 型卡车 7 辆; 方案三: A 型卡车 14 辆, B 型卡车 6 辆; 方案四: A 型卡车 15 辆, B 型卡车 5 辆.

(7 分)

(3) 设租车总费用为 W 元, 则 $W = 900n + 750 \times (20 - n) = 150n + 15000$.

$\because 150 > 0, \therefore W$ 随 n 的增大而增大.

又 $\because 12 \leq n \leq 15, \therefore$ 当 $n = 12$ 时, W 有最小值, $W_{\text{最小}} = 150 \times 12 + 15000 = 16800$.

\therefore 运费最少的租车方案是租用 A 型卡车 12 辆、B 型卡车 8 辆, 运费最少为 16800 元. (9 分)

22. 【解】(1) $\because AC = CB = 2, \angle ACB = 90^\circ,$

$$\therefore AB = 2\sqrt{2}, \angle ABC = 45^\circ,$$

$$\text{又} \because \alpha = 135^\circ, \angle MBN = \angle CBD = 45^\circ,$$

$$\therefore \angle ABM = 90^\circ.$$

$\because P$ 为 AM 中点,

$$\therefore BP = \frac{1}{2}AM.$$

$$\because CB = BM = 2, \therefore AM = \sqrt{AB^2 + MB^2} = 2\sqrt{3},$$

$$\therefore PB = \frac{1}{2}AM = \sqrt{3}. \quad (3 \text{ 分})$$

(2) 如图 1 所示, 当 C, D, M 共线时, 易知 $\angle CBM = 90^\circ, \therefore$ 旋转角 $\alpha = 90^\circ$. (7 分)

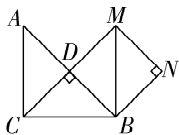


图 1

(3) $AM = 2\sqrt{3}$. (10 分)

如图 2, 连接 CM . 当 $CD \parallel BM$ 时, $\angle DCB = \angle MBN = 45^\circ$. 又 $\because \angle ABC = 45^\circ, \therefore \angle ABM = 90^\circ$,
 $\therefore AM = \sqrt{AB^2 + BM^2} = \sqrt{(2\sqrt{2})^2 + 2^2} = 2\sqrt{3}$.

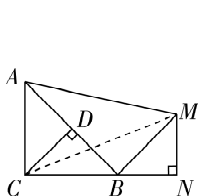


图 2

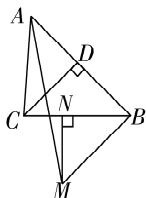


图 3

如图 3, 当 $CD \parallel BM$ 时, $\angle CDB = \angle MBA = 90^\circ$. 在 $\text{Rt} \triangle ABM$ 中,

$$AM = \sqrt{AB^2 + BM^2} = \sqrt{(2\sqrt{2})^2 + 2^2} = 2\sqrt{3}.$$

综上, $AM = 2\sqrt{3}$.

23. 【解】(1) $\because OA = 1, OC = 3OA, OB = 4OA$,

$\therefore A(-1, 0), B(4, 0), C(0, 3)$, 代入二次函数 $y =$

$$ax^2 + bx + c, \text{ 得 } \begin{cases} c = 3, \\ a - b + c = 0, \\ 16a + 4b + c = 0, \end{cases} \text{ 解得 } \begin{cases} a = -\frac{3}{4}, \\ b = \frac{9}{4}, \\ c = 3, \end{cases}$$

\therefore 抛物线解析式为 $y = -\frac{3}{4}x^2 + \frac{9}{4}x + 3$. (3 分)

(2) \because 抛物线 $y = -\frac{3}{4}x^2 + \frac{9}{4}x + 3$, 对称轴为直

$$\text{线 } x = -\frac{b}{2a} = \frac{3}{2},$$

$\therefore C(0, 3)$ 关于对称轴对称的点的坐标为 $(3, 3)$,

\therefore 结合图象得 $y \geq 3$ 时, x 的取值范围为 $0 \leq x \leq 3$.

(6 分)

(3) 设直线 BC 的解析式为 $y = kx + n$. 将 $B(4, 0), C(0, 3)$ 代入 $y = kx + n$, 得

$$\begin{cases} 4k + n = 0, \\ n = 3, \end{cases} \text{ 解得 } \begin{cases} k = -\frac{3}{4}, \\ n = 3, \end{cases} \therefore y = -\frac{3}{4}x + 3.$$

设 $P\left(\frac{3}{2}, m\right)$, 连接 BC , 过点 P 作 $PH \perp BC$, 垂足为 H , 设抛物线对称轴交 BC 于 Q .

$\because PQ \parallel OC, \therefore \angle OCB = \angle PQH. \because \angle COB =$

$$\angle PHQ = 90^\circ,$$

$$\therefore \triangle COB \sim \triangle QHP, \therefore \frac{OB}{HP} = \frac{BC}{PQ}. \because OC = 3, OB = 4, \therefore BC = 5,$$

$$\therefore \frac{4}{HP} = \frac{5}{PQ}, \therefore PH = \frac{4}{5}PQ. \quad (8 \text{ 分})$$

$$\because Q \text{ 在直线 } BC \text{ 上, 将 } x = \frac{3}{2} \text{ 代入 } y = -\frac{3}{4}x + 3, \text{ 得}$$

$$y = \frac{15}{8}, \therefore PQ = \left| \frac{15}{8} - m \right|, PH = \frac{4}{5}PQ = \left| \frac{3}{2} - \frac{4}{5}m \right|.$$

$$\because PA = \sqrt{\left(\frac{3}{2} + 1\right)^2 + m^2}, \therefore \frac{1}{5}\sqrt{\left(\frac{3}{2} + 1\right)^2 + m^2} =$$

$$\left| \frac{3}{2} - \frac{4}{5}m \right|, \text{解方程得 } m = 2 + \frac{\sqrt{6}}{3} \text{ 或 } m = 2 - \frac{\sqrt{6}}{3},$$

$$\therefore \text{点 } P \text{ 的坐标为 } \left(\frac{3}{2}, 2 + \frac{\sqrt{6}}{3}\right) \text{ 或 } \left(\frac{3}{2}, 2 - \frac{\sqrt{6}}{3}\right).$$

(10 分)