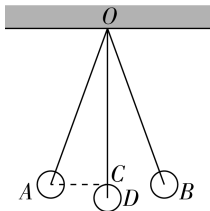


## 第二部分 期末复习突破

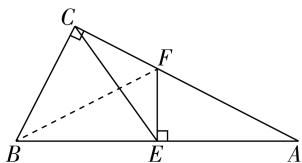
### 复习专项 (一) 基础题组

#### 上分解析

1. **C** 【解析】当  $x > 3$  时,  $-1 < y < 0$ . 故选 C.
2. **D** 【解析】根据题意设  $y = \frac{k}{x}$ , 由于点  $(0.5, 200)$  在此函数图象上,  $\therefore k = 0.5 \times 200 = 100$ ,  $\therefore y = \frac{100}{x}$ , 故选 D.
3. **A** 【解析】当  $k > 0$  时, 函数  $y = \frac{k}{x}$  的图象位于第一、三象限, 函数  $y = -kx + 2$  的图象经过第一、二、四象限; 当  $k < 0$  时, 函数  $y = \frac{k}{x}$  的图象位于第二、四象限, 函数  $y = -kx + 2$  的图象经过第一、二、三象限, 故选 A.
4. **C** 【解析】由题意可设  $A\left(\frac{a}{m}, m\right)$ ,  $B\left(\frac{b}{m}, m\right)$ , 则  $S_{\triangle ABC} = \frac{1}{2} AB \cdot y_A = \frac{1}{2} \left(\frac{a}{m} - \frac{b}{m}\right) \cdot m = 2$ ,  $\therefore a - b = 4$ . 故选 C.
5. **A** 【解析】一元二次方程  $-2(2x+1)^2 + a^2 = 0$  可化为  $-8x^2 - 8x + a^2 - 2 = 0$ .  $\because a \neq 0$ ,  $\therefore \Delta = (-8)^2 - 4 \times (-8) \times (a^2 - 2) = 64 + 32a^2 - 64 = 32a^2 > 0$ ,  $\therefore$  方程有两个不相等的实数根. 故选 A.
6. **A** 【解析】 $\because$  木条的长为  $x$  尺, 横着比门框宽 2 尺, 竖着比门框高 1.5 尺,  $\therefore$  门框的长为  $(x-1.5)$  尺, 宽为  $(x-2)$  尺,  $\therefore$  可列方程为  $(x-2)^2 + (x-1.5)^2 = x^2$ , 故选 A.
7. **C** 【解析】 $\because BE \perp AC, CD \perp AB, \therefore \angle ADC = \angle AEB = 90^\circ$ .  $\because \angle A = \angle A, \therefore \triangle ACD \sim \triangle ABE$ ,  $\therefore \frac{AC}{AB} = \frac{CD}{BE}$ ,  $\angle ACD = \angle ABE$ ,  $\therefore AC \cdot BE = AB \cdot CD$ ,  $\angle ACD = \angle ABE = \angle ABC + \angle CBE$ , 故 A、B 选项不符合题意.  $\because \angle ACB = \angle CBE + \angle E = 90^\circ + \angle CBE$ ,  $\therefore \angle ACB - \angle CBE = 90^\circ$ .  $\because \angle BCE$  与  $\angle CBE$  不一定相等,  $\therefore \angle ACB - \angle BCE = 90^\circ$  不一定成立, 故 C 选项符合题意.  $\because \angle E = 90^\circ, \therefore AB > AE$ , 即  $AD + DB > AC + CE$ , 故 D 选项不符合题意. 故选 C.
8. **D** 【解析】 $\because OA:AA' = 1:3, \therefore OA:OA' = 1:4$ .  $\because$  四边形  $ABCD$  和四边形  $A'B'C'D'$  是以点  $O$  为位似中心的位似图形,  $\therefore$  四边形  $ABCD \sim$  四边形  $A'B'C'D', AB \parallel A'B', \therefore \triangle AOB \sim \triangle A'OB', \therefore \frac{AB}{A'B'} = \frac{OA}{OA'} = \frac{1}{4}, \therefore \frac{S_{\text{四边形}ABCD}}{S_{\text{四边形}A'B'C'D'}} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$ .  $\because$  四边形  $ABCD$  的面积是 3,  $\therefore$  四边形  $A'B'C'D'$  的面积是  $3 \times 16 = 48$ , 故选 D.
9. **D** 【解析】如图, 设最低位置时的球心为  $D$ , 过  $A$  作  $AC \perp OD$  于  $C$ .  $\text{Rt} \triangle OAC$  中,  $OA = 50$  厘米,  $\angle AOC = 40^\circ \div 2 = 20^\circ, \therefore OC = OA \cdot \cos 20^\circ = 50 \cos 20^\circ, \therefore CD = OD - OC = (50 - 50 \cos 20^\circ)$  厘米. 故选 D.



(第 9 题图)



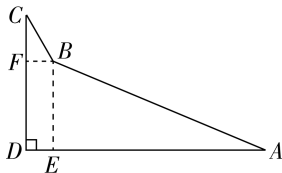
(第 10 题图)

10. **A** 【解析】连接  $BF$ , 如图.  $\because CE$  是斜边  $AB$  上的中线,  $EF \perp AB, \therefore AE = BE, \therefore EF$  是  $AB$  的垂直平分线,  $S_{\triangle AFE} = S_{\triangle BFE} = 5, \therefore AF = BF, S_{\triangle AFB} = 10, \therefore \angle FBA = \angle A$ .

$\therefore S_{\triangle AFB} = \frac{1}{2} AF \cdot BC = 10, BC = 4, \therefore AF = 5 = BF$ . 在  $\text{Rt} \triangle BCF$  中,  $BC = 4, BF = 5$ ,  
 $\therefore CF = \sqrt{5^2 - 4^2} = 3. \therefore CE = AE = BE = \frac{1}{2} AB, \therefore \angle A = \angle FBA = \angle ACE$ . 又  $\because \angle BCA = 90^\circ = \angle BEF, \therefore \angle CBF = 90^\circ - \angle BFC = 90^\circ - 2\angle A, \angle CEF = 90^\circ - \angle BEC = 90^\circ - 2\angle A, \therefore \angle CEF = \angle FBC, \therefore \sin \angle CEF = \sin \angle FBC = \frac{CF}{BF} = \frac{3}{5}$ , 故选 A.

11.4 【解析】 $\because$  斜坡的坡度  $i = 1 : \sqrt{3}, \therefore BC : AC = 1 : \sqrt{3}. \therefore AC = 2\sqrt{3}$  米,  $\therefore BC = 2$  米,  $\therefore AB = \sqrt{AC^2 + BC^2} = \sqrt{(2\sqrt{3})^2 + 2^2} = 4$  (米), 故答案为 4.

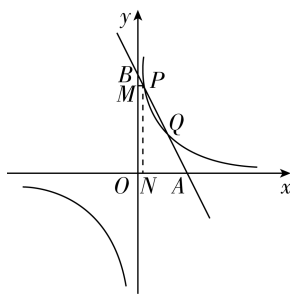
12.  $(150\sqrt{3} + 500)$  【解析】如图, 过点  $B$  作  $BE \perp AD$  于点  $E$ , 作  $BF \perp CD$  于点  $F$ , 则四边形  $FDEB$  为矩形,  $\therefore DE = FB, DF = BE$ . 在  $\text{Rt} \triangle BFC$  中,  $BC = 300$  m,  $\angle CBF = 60^\circ$ , 则  $CF = BC \cdot \sin \angle CBF = 300 \times \frac{\sqrt{3}}{2} = 150\sqrt{3}$  (m). 设  $BE = 5x$  m.  $\because$  斜坡  $AB$  的坡比为  $5 : 12, \therefore AE = 12x$  m. 由勾股定理得  $AB^2 = BE^2 + AE^2$ , 即  $1300^2 = (5x)^2 + (12x)^2$ , 解得  $x = 100$  (负值舍去),  $\therefore BE = 500$  m, 则  $DF = BE = 500$  m,  $\therefore CD = CF + DF = (150\sqrt{3} + 500)$  m, 故答案为  $(150\sqrt{3} + 500)$ .



13.3 【解析】 $\because$  四边形  $ABCD$  是平行四边形,  $\therefore AD \parallel BC, AD = BC. \therefore AF = 2FD, \therefore AF = \frac{2}{3} AD = \frac{2}{3} BC, DF = \frac{1}{3} AD = \frac{1}{3} BC. \therefore AF \parallel BC, \therefore \triangle AEF \sim \triangle CEB, \therefore \frac{EF}{EB} = \frac{AF}{CB}$ , 即  $\frac{EF}{2} = \frac{2}{3}, \therefore EF = \frac{4}{3}. \therefore DF \parallel BC, \therefore \triangle GFD \sim \triangle GBC, \therefore \frac{GF}{GB} = \frac{DF}{CB}$ , 即  $\frac{GF}{GF + \frac{4}{3} + 2} = \frac{1}{3}, \therefore GF = \frac{5}{3}, \therefore EG = EF + GF = \frac{4}{3} + \frac{5}{3} = 3$ . 故答案为 3.

14.  $302(1+x)^2 = 503$  【解析】 $\because$  第一个月新建了 302 个充电桩, 该市新建智能充电桩个数的月平均增长率为  $x, \therefore$  第二个月新建了  $302(1+x)$  个充电桩,  $\therefore$  第三个月新建了  $302(1+x)^2$  个充电桩.  $\because$  第三个月新建了 503 个充电桩,  $\therefore 302(1+x)^2 = 503$ , 故答案为  $302(1+x)^2 = 503$ .

15.10 【解析】如图, 过  $P$  向  $x$  轴、 $y$  轴作垂线, 垂足分别为  $N, M, \therefore$  易得  $\angle BPM = \angle BAO. \because P$  点在双曲线  $y = \frac{4}{x}$  上,  $\therefore$  设  $P\left(m, \frac{4}{m}\right)$ , 代入  $y = -2x + b$  得  $\frac{4}{m} = -2m + b, \therefore m(b - 2m) = 4. \therefore$  一次函数  $y = -2x + b$  ( $b$  为常数) 的图象与  $x$  轴、 $y$  轴分别交于点  $A, B, \therefore$  令  $x = 0$ , 则  $y = b, \therefore B(0, b),$  令  $y = 0$ , 则  $x = \frac{b}{2},$



$\therefore A\left(\frac{b}{2}, 0\right), \therefore OA = \frac{b}{2}, OB = b. \therefore P\left(m, \frac{4}{m}\right), \therefore OM = \frac{4}{m}, ON = m, \therefore AN = \frac{b}{2} - m, PM = m. \therefore$  易得  $PM \parallel OA, \therefore \triangle PMB \sim \triangle AOB, \therefore \frac{BM}{BO} = \frac{PM}{OA}, \therefore \frac{BM}{PM} = \frac{OB}{OA} = 2, \therefore BM = 2MP$ , 同理可得  $PN = 2AN, \therefore AP = \sqrt{AN^2 + PN^2} = \sqrt{5} AN, BP = \sqrt{BM^2 + MP^2} = \sqrt{5} PM, \therefore AP \cdot BP = 5AN \cdot PM = 5\left(\frac{b}{2} - m\right) \times m. \because m(b - 2m) = 4, \therefore AP \cdot BP = 10$ . 故答案为 10.

16. 直角三角形 【解析】 $\because (2\sin A - 1)^2 + \sqrt{\cos B - \frac{1}{2}} = 0, \therefore 2\sin A - 1 = 0, \cos B - \frac{1}{2} = 0, \therefore \sin A = \frac{1}{2}, \cos B = \frac{1}{2}, \therefore \angle A = 30^\circ, \angle B = 60^\circ, \therefore \angle C = 90^\circ, \therefore \triangle ABC$  是直

角三角形.

17.  $y = \frac{1}{4x}$  【解析】连接  $OB$ .  $\because$  四边形  $OABC$  为矩形, 且点  $B$  的坐标为  $(4, 1)$ ,  $\therefore AB = OC = 1, OA = BC = 4$ . 由旋转的性质得  $OB' = OB, A'B' = AB = 1, OA' = OA = 4$ .  
 $\therefore \angle COD = \angle B'OA', \angle A' = \angle OCD = 90^\circ, \therefore \triangle COD \sim \triangle A'OB', \therefore \frac{OC}{OA'} = \frac{CD}{A'B'}$ , 即  
 $\frac{1}{4} = \frac{CD}{1}$ , 解得  $CD = \frac{1}{4}, \therefore D\left(\frac{1}{4}, 1\right)$ ,  $\therefore$  经过点  $D$  的反比例函数图象的表达式是  
 $y = \frac{1}{4x}$ , 故答案为  $y = \frac{1}{4x}$ .

18.  $2\sqrt{5}$  【解析】作  $NP \perp AB$  于点  $P$ , 如图. 在  $\text{Rt} \triangle ACB$

中, 由勾股定理得  $AB = \sqrt{AC^2 + BC^2} = \sqrt{10^2 + 5^2} =$

$5\sqrt{5}$ . 设  $AM$  长为  $x$ , 则  $BM = 5\sqrt{5} - x$ .  $\therefore \tan \angle MAN = \frac{MN}{AN} =$

$\frac{1}{2}, \therefore AN = 2MN, \therefore AM = \sqrt{AN^2 + NM^2} = \sqrt{5}MN, \therefore MN = \frac{\sqrt{5}}{5}AM = \frac{\sqrt{5}}{5}x, AN = 2MN =$

$\frac{2\sqrt{5}}{5}x$ . 同理, 在  $\text{Rt} \triangle ANP$  中, 可得  $NP = \frac{\sqrt{5}}{5}AN = \frac{2}{5}x, AP = 2NP = \frac{4}{5}x. \therefore O$  为  $BM$  中

点,  $\therefore BO = \frac{1}{2}BM = \frac{5\sqrt{5} - x}{2}, \therefore AO = AB - BO = \frac{5\sqrt{5} + x}{2}, \therefore OP = AO - AP = \frac{5\sqrt{5} + x}{2} - \frac{4}{5}x =$

$\frac{25\sqrt{5} - 3x}{10}$ . 在  $\text{Rt} \triangle ONP$  中, 由勾股定理得  $ON^2 = OP^2 + NP^2$ , 即  $ON^2 = \left(\frac{25\sqrt{5} - 3x}{10}\right)^2 +$

$\left(\frac{2}{5}x\right)^2 = \frac{1}{100}(25x^2 - 150\sqrt{5}x + 3125) = \frac{1}{4}(x^2 - 6\sqrt{5}x + 125) = \frac{1}{4}(x - 3\sqrt{5})^2 + 20,$

$\therefore$  当  $x = 3\sqrt{5}$  时,  $ON^2$  取得最小值 20,  $\therefore ON$  长的最小值为  $2\sqrt{5}$ . 故答案为  $2\sqrt{5}$ .

19. 【解】(1)  $\because x^2 + 2x = 1, \therefore x^2 + 2x + 1 = 2, \therefore (x+1)^2 = 2, \therefore x+1 = \sqrt{2}$  或  $x+1 = -\sqrt{2}$ , 解得  
 $x_1 = -1 + \sqrt{2}, x_2 = -1 - \sqrt{2}$ .

(2)  $4x(2x+1) = 3(2x+1), 4x(2x+1) - 3(2x+1) = 0, (4x-3)(2x+1) = 0, \therefore 4x-3 =$

$0$  或  $2x+1=0$ , 解得  $x_1 = \frac{3}{4}, x_2 = -\frac{1}{2}$ .

20. 【解】根据题意得  $k+2 \neq 0$  且  $\Delta = (-2k)^2 - 4(k+2)(k-1) \geq 0$ , 解得  $k \leq 2$  且  $k \neq$

$-2. \therefore$  关于  $x$  的方程  $(k+2)x^2 - 2kx + k-1 = 0$  的两个实数根为  $\alpha, \beta, \therefore \alpha + \beta = \frac{2k}{k+2}$ .

$\therefore \alpha^2 = \beta^2, \therefore \alpha = \beta$  或  $\alpha = -\beta$ . 当  $\alpha = \beta$  时,  $\Delta = (-2k)^2 - 4(k+2)(k-1) = 0$ , 解得  $k = 2$ ;

当  $\alpha = -\beta$  时,  $\alpha + \beta = \frac{2k}{k+2} = 0$ , 解得  $k = 0$ . 综上所述,  $k$  的值为 0 或 2.

21. 【解】(1)  $\because$  点  $A(m, 2)$  在反比例函数  $y = \frac{k}{x} (k > 0, x > 0)$  的图象上,  $\therefore k = 2m, \therefore y =$

$\frac{2m}{x}$ .  $\because$  点  $B$  在反比例函数  $y = \frac{k}{x} (k > 0, x > 0)$  的图象上, 点  $B$  的横坐标是 4, 过点  $B$

作  $BC \perp x$  轴于点  $C, \therefore y = \frac{2m}{4} = \frac{1}{2}m, \therefore BC = \frac{1}{2}m$ . 故答案为  $\frac{1}{2}m$ .

(2)  $S = \frac{1}{2} \times \frac{1}{2}m \times (4-m) = -\frac{1}{4}m^2 + m (0 < m < 4)$ .

(3) 由  $S = -\frac{1}{4}m^2 + m = -\frac{1}{4}(m-2)^2 + 1, \therefore$  当  $m = 2$  时,  $\triangle ABC$  的面积  $S$  最大,  $\therefore$  当

$\triangle ABC$  的面积  $S$  最大时, 反比例函数的表达式为  $y = \frac{4}{x}$ .

22. 【解】(1)  $\because$  反比例函数  $y = \frac{2}{x}$  的图象与一次函数  $y = kx + b$  的图象交于点  $A, B$ , 点

$A, B$  的横坐标分别为  $1, -2$ ,  $\therefore A(1, 2), B(-2, -1)$ . 把  $A, B$  的坐标代入  $y = kx + b$

$$\text{得} \begin{cases} k+b=2, \\ -2k+b=-1, \end{cases} \text{解得} \begin{cases} k=1, \\ b=1, \end{cases} \therefore \text{一次函数的表达式为 } y=x+1.$$

(2)  $-2 < x < 0$ . 由图象可知, 当  $-2 < x < 0$  时,  $y < -1$ .

(3)  $\because$  一次函数  $y = x + 1$ ,  $\therefore D(-1, 0)$ .  $\because A(1, 2)$ ,  $\therefore S_{\triangle ODA} = \frac{1}{2} \times 2 \times 1 = 1$ ,  $\therefore S_{\triangle BDP} =$

$\frac{1}{2} S_{\triangle ODA} = \frac{1}{2}$ . 作  $PN \perp x$  轴于  $N$ ,  $BM \perp x$  轴于  $M$ . 设点  $P$  的坐标为  $(x, \frac{2}{x})$ , 其中  $x <$

$0$ ,  $\therefore ON = -x$ ,  $PN = -\frac{2}{x}$ . 当  $P$  在直线  $AB$  的下方时, 如图(1), 则  $S_{\triangle BDP} = S_{\text{梯形} BMNP} +$

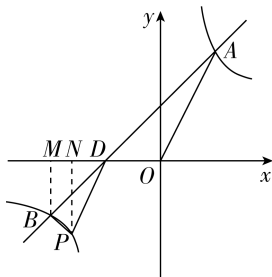
$$S_{\triangle NDP} - S_{\triangle BDM} = \frac{1}{2} \times \left(1 - \frac{2}{x}\right) \times (2+x) + \frac{1}{2} \times (-x-1) \times \left(-\frac{2}{x}\right) - \frac{1}{2} \times (2-1) \times 1 = \frac{1}{2}, \text{解}$$

得  $x = -\sqrt{2}$  (正值已舍去),  $\therefore P(-\sqrt{2}, -\sqrt{2})$ . 当  $P$  在直线  $AB$  的上方时, 如图

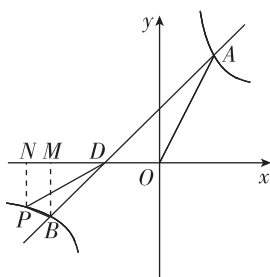
(2), 则  $S_{\triangle BDP} = S_{\text{梯形} BMNP} + S_{\triangle BDM} - S_{\triangle PDN} = \frac{1}{2} \times \left(1 - \frac{2}{x}\right) \times (-x-2) + \frac{1}{2} \times (2-1) \times 1 -$

$$\frac{1}{2} \times (-x-1) \times \left(-\frac{2}{x}\right) = \frac{1}{2}, \text{解得 } x = -1 - \sqrt{3} \text{ (正值已舍去)}, \therefore P(-1 - \sqrt{3}, 1 -$$

$\sqrt{3})$ . 综上, 点  $P$  的坐标为  $(-\sqrt{2}, -\sqrt{2})$  或  $(-1 - \sqrt{3}, 1 - \sqrt{3})$ .



图(1)



图(2)

23. (1) 【证明】 $\because$  四边形  $ABCD$  是菱形,  $\therefore \angle ABQ = \angle CBQ, AB = BC, AB \parallel CD$ . 在

$$\triangle ABQ \text{ 和 } \triangle CBQ \text{ 中, } \begin{cases} AB=CB, \\ \angle ABQ = \angle CBQ, \\ BQ=BQ, \end{cases} \therefore \triangle ABQ \cong \triangle CBQ \text{ (SAS)}, \therefore \angle BAQ = \angle BCQ.$$

$\because AB \parallel CD, \therefore \angle BAQ = \angle N, \therefore \angle BCQ = \angle N. \therefore \angle CQM = \angle NQC,$

$$\therefore \triangle CQM \sim \triangle NQC, \therefore \frac{CQ}{NQ} = \frac{QM}{CQ}.$$

(2) 【解】 $\because QN = 8, MN = 6, \therefore QM = 2$ . 由(1)知,  $\frac{CQ}{NQ} = \frac{QM}{CQ}, \therefore \frac{CQ}{8} = \frac{2}{CQ}, \therefore CQ = 4$ .

由(1)知,  $\triangle ABQ \cong \triangle CBQ, \therefore AQ = CQ = 4, \therefore AM = AQ + QM = 4 + 2 = 6$ . 在  $\text{Rt} \triangle CQM$

中,  $CM = \sqrt{CQ^2 - QM^2} = \sqrt{16 - 4} = 2\sqrt{3}$ . 设  $BM = x$ , 则  $AB = BC = BM + CM = x + 2\sqrt{3}$ . 在

$\text{Rt} \triangle ABM$  中,  $AB^2 = AM^2 + BM^2$ , 即  $(x + 2\sqrt{3})^2 = 6^2 + x^2$ , 解得  $x = 2\sqrt{3}$ , 即  $BM = 2\sqrt{3}$ ,

$\therefore BM = CM. \because AM \perp BC, \therefore$  易得  $\triangle ABC$  是等边三角形. 又  $\because$  四边形  $ABCD$  是菱形,  $\therefore AC \perp BD, \therefore BD = 2AM = 12$ .

24. 【解】(1)  $1\ 000 - 68 - 510 - 177 = 245$  (人). 故  $a$  的值为 245.

(2) ①最适合的统计图是扇形统计图, 故答案为扇形统计图.

②市民偶尔执行“荷式开门法”的人数最多, 故抽取的市民中 C 类别人数占比最

大. 其所在扇形对应圆心角的度数为  $\frac{510}{1\ 000} \times 360^\circ = 183.6^\circ$ .

$$(3) 20 \times \frac{177}{1\,000} = 3.54 (\text{万人}) = 35\,400 (\text{人}),$$

∴ 估计该市活动前从不执行“荷式开门法”的总人数是 35 400 人.

(4) 小明分析数据的方法不合理,理由如下:

$$\text{宣传活动后从不执行“荷式开门法”所占百分比为 } \frac{178}{896+702+224+178} \times 100\% =$$

8.9%,

$$\text{宣传活动前从不执行“荷式开门法”所占百分比为 } \frac{177}{1\,000} \times 100\% = 17.7\%,$$

而  $8.9\% < 17.7\%$ , 因此公益团队开展的宣传活动有效果.

## 复习专项 (二) 中等题组

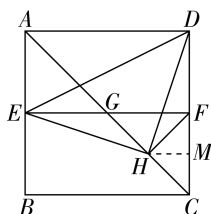
### 上分解析

1. **A** 【解析】根据题图得  $k > 0, b < 0, \therefore kb < 0. \therefore \Delta = 1^2 - 4kb > 0, \therefore$  方程有两个不相等的实数根. 故选 A.

2. **C** 【解析】如图, 过点  $P$  作  $PK \perp BE$  于点  $K. \therefore \triangle ABE$  为直角三角形,  $\therefore \angle AEB = 90^\circ. \therefore PK \perp BE, \therefore \angle PKF = 90^\circ, \therefore PK \parallel AE, \therefore \triangle BKP \sim \triangle BEA, \therefore \frac{PK}{AE} = \frac{BK}{BE} = \frac{BP}{BA}. \therefore P$  为  $AB$  的中点,  $\therefore \frac{PK}{AE} = \frac{BK}{BE} = \frac{1}{2}$ . 根据题意可得  $\angle EFG = \angle FGH = 90^\circ, \therefore BE \parallel GD, \therefore \angle PFK = \angle FQG$ . 在  $\triangle PFK$  和  $\triangle FQG$  中,  $\angle PFK = \angle FQG, \angle PKF = \angle FGQ = 90^\circ, \therefore \triangle PFK \sim \triangle FQG, \therefore \frac{FK}{QG} = \frac{PK}{FG}$ . 设  $FG = GH = EH = EF = a, AH = BE = b$ , 则  $QG = \frac{1}{2}a, AE = b - a. \therefore \frac{PK}{AE} = \frac{BK}{BE} = \frac{1}{2}, \therefore PK = \frac{1}{2}(b - a), EK = \frac{1}{2}b, \therefore FK = EK - EF = \frac{1}{2}b - a. \therefore \frac{FK}{QG} = \frac{PK}{FG}, \therefore \frac{\frac{1}{2}b - a}{\frac{1}{2}a} = \frac{\frac{1}{2}(b - a)}{a}, \therefore b = 3a$ . 在  $\text{Rt} \triangle ABE$  中,  $AE = b - a, BE = b$ , 由勾股定理得  $AB = \sqrt{(b - a)^2 + b^2} = \sqrt{(3a - a)^2 + (3a)^2} = \sqrt{13}a, \therefore \frac{AB}{EF} = \frac{\sqrt{13}a}{a} = \sqrt{13}$ , 故选 C.

3. **C** 【解析】因为抽查了 20 名学生的视力, 数据在 4.85~5.15 这一小组的频数为 8, 所以该校八年级学生视力在 4.85~5.15 范围内的人数有  $500 \times \frac{8}{20} = 200$  (人). 故选 C.

4. **C** 【解析】①  $\because$  四边形  $ABCD$  为正方形,  $EF \parallel AD, \therefore$  易得四边形  $AEFD$  为矩形,  $\angle ACD = 45^\circ, \angle GFC = 90^\circ, \therefore EF = AD = CD, \triangle CFG$  为等腰直角三角形,  $\therefore GF = FC. \therefore EG = EF - GF, DF = CD - FC, \therefore EG = DF$ , 故①正确. ②  $\because \triangle CFG$  为等腰直角三角形,  $H$  为  $CG$  的中点,  $\therefore FH = CH, \angle GFH = \frac{1}{2} \angle GFC = 45^\circ = \angle HCD$ . 在  $\triangle EHF$  和  $\triangle DHC$  中,  $\begin{cases} EF = CD, \\ \angle EFH = \angle DCH, \\ FH = CH, \end{cases} \therefore \triangle EHF \cong \triangle DHC (SAS), \therefore \angle HEF = \angle HDC, \therefore \angle AEH + \angle ADH = \angle AEF + \angle HEF + \angle ADF - \angle HDC = \angle AEF + \angle ADF = 180^\circ$ , 故②正确. ③  $\because DF < EF, \angle DFE = 90^\circ, \therefore \angle FED < 45^\circ, \therefore \angle FED \neq \angle GFH, \therefore FH$  与  $DE$  不平行, 故③错误. ④  $\because \frac{AE}{AB} = \frac{2}{3}, \therefore AE = 2BE. \therefore \triangle CFG$  为等腰直角三角形,  $H$  为  $CG$  的中点,  $\therefore FH = GH, \angle FGH = 90^\circ, \therefore \angle EGH = \angle FGH + \angle HFG = 90^\circ + \angle HFG = \angle HFD$ . 在  $\triangle EGH$  和  $\triangle DFH$  中,  $\begin{cases} EG = DF, \\ \angle EGH = \angle HFD, \\ GH = FH, \end{cases} \therefore \triangle EGH \cong \triangle DFH (SAS), \therefore \angle EHG = \angle DHF, EH = DH, \therefore \angle DHE = \angle EHG + \angle DHG = \angle DHF + \angle DHG = \angle FGH = 90^\circ, \therefore \triangle EHD$  为等腰直角三角形. 过  $H$  点作  $HM \perp CD$  于  $M$  点, 如图所示. 设  $HM = x$ , 则  $CF = 2x, \therefore DF = 2FC = 4x, \therefore DM = 5x, CD = 6x, \therefore DH =$

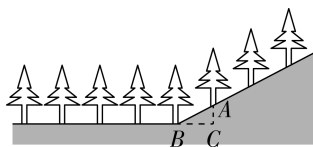


$\sqrt{26}x$ , 则  $S_{\triangle DHC} = \frac{1}{2} \times HM \times DC = \frac{1}{2} \cdot x \cdot 6x = 3x^2$ ,  $S_{\triangle EDH} = \frac{1}{2} \times DH^2 = \frac{1}{2} \times (\sqrt{26}x)^2 =$

$13x^2$ ,  $\therefore \frac{S_{\triangle DHC}}{S_{\triangle EDH}} = \frac{3x^2}{13x^2} = \frac{3}{13}$ , 故④正确. 故其中结论正确的有 3 个. 故选 C.

5.  $\frac{3}{4}$  【解析】如图, 根据题意得  $AB = 5$  米,  $BC =$

4 米,  $\therefore AC = \sqrt{AB^2 - BC^2} = 3$  米,  $\therefore \frac{AC}{BC} = \frac{3}{4}$ , 即此山

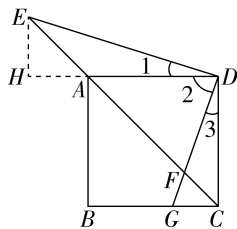


坡的坡度为  $\frac{3}{4}$ . 故答案为  $\frac{3}{4}$ .

6. ①②③ 【解析】 $\because \angle A = \angle A$ ,  $\angle AED = \angle B$ ,  $\therefore$  根据“两角分别相等的两个三角形相似”可证  $\triangle ADE \sim \triangle ACB$ , 故①满足题意;  $\because \angle A = \angle A$ ,  $\angle ADE = \angle C$ ,  $\therefore$  根据“两角分别相等的两个三角形相似”可证  $\triangle ADE \sim \triangle ACB$ , 故②满足题意;  $\because \angle A = \angle A$ ,  $AD \cdot AB = AE \cdot AC$ ,  $\therefore AD : AC = AE : AB$ ,  $\therefore$  根据“两边成比例且夹角相等的两个三角形相似”可证  $\triangle ADE \sim \triangle ACB$ , 故③满足题意;  $\because AD : AC = DE : BC$ , 而  $\angle ADE$  与  $\angle C$  不一定相等,  $\therefore$  无法证明  $\triangle ADE$  与  $\triangle ACB$  相似, 故④不满足题意, 综上可得①②③符合题意. 故答案为①②③.

7.  $\frac{5\sqrt{5}}{2}$  【解析】如图, 过点  $E$  作  $EH \perp AD$ , 交  $DA$  延长线于

$H$ ,  $\therefore \angle H = 90^\circ$ . 在正方形  $ABCD$  中,  $AB = BC = CD = AD$ ,  $\angle BAD = \angle B = \angle BCD = \angle ADC = 90^\circ$ ,  $\therefore \angle 2 + \angle 3 = 90^\circ$ ,  $\angle H = \angle BCD$ .  $\because DE \perp DG$ ,  $\therefore \angle EDG = 90^\circ$ ,  $\therefore \angle 2 + \angle 1 =$



$90^\circ$ ,  $\therefore \angle 1 = \angle 3$ ,  $\therefore \triangle DEH \sim \triangle DGC$ ,  $\therefore \frac{EH}{GC} = \frac{DH}{DC}$ ,  $\therefore \frac{GC}{BG} =$

$\frac{1}{2}$ ,  $\therefore$  设  $GC = x (x > 0)$ , 则  $BG = 2x$ ,  $DC = BC = 3x$ ,  $\therefore \frac{EH}{x} = \frac{DH}{3x}$ ,  $\therefore DH = 3EH$ .  $\because AC$  是正方形  $ABCD$  对角线,  $\therefore \angle DAC = 45^\circ$ ,  $\therefore \angle EAH = \angle DAC = 45^\circ$ ,  $\therefore \angle HEA = 45^\circ$ ,

$\therefore EH = HA$ .  $\because AE = 5$ ,  $\therefore EH^2 + HA^2 = AE^2 = 25$ ,  $\therefore EH = HA = \frac{5\sqrt{2}}{2}$ ,  $\therefore DH = \frac{15\sqrt{2}}{2}$ ,  $\therefore BC =$

$CD = AD = DH - HA = 5\sqrt{2}$ . 由  $\frac{GC}{BG} = \frac{1}{2}$  得  $GC = \frac{1}{3}BC = \frac{5\sqrt{2}}{3}$ ,  $\therefore DG = \sqrt{CD^2 + GC^2} =$

$\frac{10\sqrt{5}}{3}$ . 在正方形  $ABCD$  中,  $AD \parallel BC$ ,  $\therefore \triangle ADF \sim \triangle CGF$ ,  $\therefore \frac{DF}{GF} = \frac{AD}{CG} = \frac{BC}{CG} = 3$ ,  $\therefore DF =$

$3GF$ ,  $\therefore DF = \frac{3}{4}DG = \frac{5\sqrt{5}}{2}$ , 故答案为  $\frac{5\sqrt{5}}{2}$ .

8. 【解】(1)  $\because A(1, 6)$  是反比例函数  $y = \frac{k}{x} (x > 0)$  的图象上的点,  $\therefore k = 1 \times 6 = 6$ ,  $\therefore$  反

比例函数的表达式为  $y = \frac{6}{x}$ .

(2) 把  $B(3, m)$  代入  $y = \frac{6}{x}$ , 得  $m = \frac{6}{3} = 2$ ,  $\therefore B(3, 2)$ . 设  $P$  点的坐标为  $(x, 0)$ .  $\because$  线段  $AB$  的垂直平分线交  $x$  轴于点  $P$ ,  $\therefore PA = PB$ ,  $\therefore (x-1)^2 + 6^2 = (x-3)^2 + 2^2$ , 解得  $x = -6$ ,  $\therefore$  点  $P$  的坐标为  $(-6, 0)$ .

9. 【解】(1) 根据表中数据可知,  $vt = 30$ ,  $\therefore v = \frac{30}{t}$ ,  $\therefore$  平均速度  $v$  (千米/时) 关于骑行时

间  $t$  (时) 的函数表达式为  $v = \frac{30}{t}$ .

(2) 骑行爱好者不能在上午 9:10 之前到达上海蟠龙天地, 理由:  $\because$  从上午 8:30 到上午 9:10, 骑行者用时 40 分钟, 即  $\frac{2}{3}$  小时, 当  $t = \frac{2}{3}$  时,  $v = \frac{30}{\frac{2}{3}} = 45$ .  $\therefore$  平均速度不





## 复习专项 (三) 重难题组

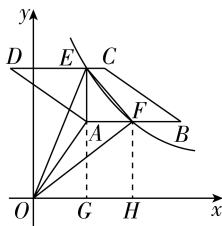
### 上分解析

1. A 【解析】如图. 延长  $EA$  交  $x$  轴于点  $G$ , 过点  $F$  作  $FH \perp x$  轴于点  $H$ .  $\because AB \parallel x$  轴,  $AE \perp CD$ ,  $AB \parallel CD$ ,  $\therefore AG \perp x$  轴.

$\because AO \perp AD$ ,  $\therefore \angle DAE + \angle OAG = 90^\circ$ .  $\because AE \perp CD$ ,  $\therefore \angle DAE + \angle D = 90^\circ$ ,  $\therefore \angle D = \angle OAG$ . 在  $\triangle DAE$  和  $\triangle AOG$  中,

$$\begin{cases} \angle DEA = \angle AGO = 90^\circ, \\ \angle D = \angle OAG, \end{cases} \therefore \triangle DAE \cong \triangle AOG \text{ (AAS)}, \therefore DE =$$

$$AD = OA,$$



$AG, AE = OG$ .  $\because$  四边形  $ABCD$  是菱形,  $DE = 4CE$ ,  $\therefore AD = CD = \frac{5}{4}DE$ , 设  $DE = 4a$ , 则

$$AD = OA = 5a, \therefore OG = AE = \sqrt{AD^2 - DE^2} = \sqrt{(5a)^2 - (4a)^2} = 3a, \therefore EG = AE + AG =$$

$$7a, \therefore E(3a, 7a). \because \text{反比例函数 } y = \frac{k}{x} (x > 0) \text{ 的图象经过点 } E, \therefore k = 21a^2.$$

$\because AG \perp GH, FH \perp GH, AF \perp AG$ ,  $\therefore$  四边形  $AGHF$  为矩形,  $\therefore HF = AG = 4a$ .  $\because$  点  $F$  在

$$\text{反比例函数 } y = \frac{k}{x} (x > 0) \text{ 的图象上}, \therefore x = \frac{21a^2}{4a} = \frac{21}{4}a, \therefore F\left(\frac{21}{4}a, 4a\right), \therefore OH =$$

$$\frac{21}{4}a, \therefore GH = OH - OG = \frac{9}{4}a. \therefore S_{\triangle OEF} = S_{\triangle OEG} + S_{\text{梯形 } EGHF} - S_{\triangle OFH}, S_{\triangle OEF} = \frac{11}{8}, \therefore \frac{1}{2} \times OG \times$$

$$EG + \frac{1}{2} (EG + FH) \times GH - \frac{1}{2} OH \times HF = \frac{11}{8}, \therefore \frac{1}{2} \times 3a \times 7a + \frac{1}{2} \times (7a + 4a) \times \frac{9}{4}a - \frac{1}{2} \times$$

$$21a^2 = \frac{11}{8}, \text{解得 } a^2 = \frac{1}{9}, \therefore k = 21a^2 = 21 \times \frac{1}{9} = \frac{7}{3}, \text{故选 A.}$$

2. C 【解析】 $\because x^2 - xy + 4y^2 = 4$ ,  $\therefore x^2 + 4y^2 = xy + 4$ ,  $\therefore u = x^2 + xy + 4y^2 = 2xy + 4$ .  $\because 5xy =$

$$4xy + (x^2 + 4y^2 - 4) = (x + 2y)^2 - 4 \geq -4, \text{当且仅当 } x = -2y, \text{即 } x = -\frac{2\sqrt{10}}{5}, y = \frac{\sqrt{10}}{5} \text{ 或}$$

$$x = \frac{2\sqrt{10}}{5}, y = -\frac{\sqrt{10}}{5} \text{ 时等号成立. } \therefore xy \text{ 的最小值为 } -\frac{4}{5}, \therefore u = x^2 + xy + 4y^2 = 2xy + 4$$

$$\text{的最小值为 } \frac{12}{5}, \text{即 } m = \frac{12}{5}. \therefore 3xy = 4xy - (x^2 + 4y^2 - 4) = 4 - (x - 2y)^2 \leq 4, \text{当且仅当 } x =$$

$$2y, \text{即 } x = \frac{2\sqrt{6}}{3}, y = \frac{\sqrt{6}}{3} \text{ 或 } x = -\frac{2\sqrt{6}}{3}, y = -\frac{\sqrt{6}}{3} \text{ 时等号成立. } \therefore xy \text{ 的最大值为 } \frac{4}{3}, \therefore u =$$

$$x^2 + xy + 4y^2 = 2xy + 4 \text{ 的最大值为 } \frac{20}{3}, \text{即 } M = \frac{20}{3}. \therefore M + m = \frac{20}{3} + \frac{12}{5} = \frac{136}{15}.$$

3. B 【解析】 $\because \triangle ABC$  为等边三角形,  $\triangle ABD$  为等腰直角三角形,  $\angle BAD = 90^\circ$ ,

$$\therefore \angle BAC = 60^\circ, AC = AB = AD, \angle ADB = \angle ABD = 45^\circ, \therefore \angle CAD = 150^\circ, \therefore \angle ADC =$$

$$15^\circ, \text{故①正确. } \because AE \perp BD, \text{即 } \angle AED = 90^\circ, \therefore \angle DAE = 45^\circ, \therefore \angle AFG = \angle ADC +$$

$$\angle DAE = 60^\circ, \angle FAG = 45^\circ, \therefore \angle AGF = 75^\circ. \text{由 } \angle AFG \neq \angle AGF \text{ 知 } AF \neq AG, \text{故②错}$$

$$\text{误. 记 } AH \text{ 与 } CD \text{ 的交点为 } P, \text{由 } AH \perp CD \text{ 且 } \angle AFG = 60^\circ \text{ 知 } \angle FAP = 30^\circ, \text{则 } \angle BAH =$$

$$\angle ADC = 15^\circ. \text{在 } \triangle ADF \text{ 和 } \triangle BAH \text{ 中, } \therefore \begin{cases} \angle ADF = \angle BAH, \\ DA = AB, \\ \angle DAF = \angle ABH = 45^\circ, \end{cases} \therefore \triangle ADF \cong$$

$$\triangle BAH \text{ (ASA)}, \therefore DF = AH, \text{故③正确. } \because \angle AFG = \angle CBG = 60^\circ, \angle AGF = \angle CGB,$$

$$\therefore \triangle AFG \sim \triangle CBG, \text{故④正确. 在 Rt } \triangle APF \text{ 中, 设 } PF = x, \text{则 } AF = 2x, AP =$$

$$\sqrt{AF^2 - PF^2} = \sqrt{3}x, \text{设 } EF = a. \because \triangle ADF \cong \triangle BAH, \therefore BH = AF = 2x. \triangle ABE \text{ 中,}$$

$$\therefore \angle AEB = 90^\circ, \angle ABE = 45^\circ, \therefore BE = AE = AF + EF = a + 2x, \therefore EH = BE - BH = a + 2x -$$

$2x = a$ .  $\because \angle APF = \angle AEH = 90^\circ$ ,  $\angle FAP = \angle HAE$ ,  $\therefore \triangle PAF \sim \triangle EAH$ ,  $\therefore \frac{PF}{EH} = \frac{AP}{AE}$ , 即

$\frac{x}{a} = \frac{\sqrt{3}x}{a+2x}$ , 整理, 得  $2x^2 = (\sqrt{3}-1)ax$ , 由  $x \neq 0$  得  $2x = (\sqrt{3}-1)a$ , 即  $AF = (\sqrt{3}-1)EF$ , 故⑤正确. 故选 B.

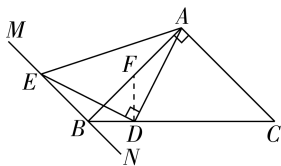
4. B 【解析】 $\because$  四边形  $ABCD$  是矩形,  $BC = 2AB$ ,  $\therefore AB = CD$ ,  $AD = BC$ ,  $\therefore AD = 2CD$ . 设  $CG = x$ ,  $HG = y$ .  $\because \triangle AHD \cong \triangle CFB$ ,  $\triangle ABE \cong \triangle CDG$ , 且这四个三角形均为直角三角形,  $\therefore \angle AHD = \angle DGC = \angle ADC = 90^\circ$ ,  $\therefore \angle DAH + \angle ADH = \angle ADH + \angle CDG = 90^\circ$ ,  $\therefore \angle CDG = \angle DAH$ ,  $\therefore \triangle ADH \sim \triangle DCG$ ,  $\therefore \frac{DH}{CG} = \frac{AH}{DG} = \frac{AD}{CD} = 2$ ,  $\therefore DH = 2x$ ,  $\therefore DG = 2x + y$ ,  $AH = 4x + 2y$ ,  $EH = 3x + 2y$ .  $\because \angle CHG = \alpha$ ,  $\angle CDG = \beta$ ,  $\tan \beta = \tan^2 \alpha$ ,  $\therefore \frac{x}{2x+y} = \frac{x^2}{y^2}$ , 即  $2x^2 + xy = y^2$ ,  $\therefore y^2 - xy - 2x^2 = 0$ ,  $\therefore (y-2x)(y+x) = 0$ .  $\because y+x \neq 0$ ,  $\therefore y = 2x$ ,  $\therefore DG = 4x$ ,  $DC = \sqrt{17}x$ ,  $EH = 3x + 2y = 7x$ ,  $\therefore AD = 2\sqrt{17}x$ ,  $\therefore \frac{S_{\text{矩形}EFGH}}{S_{\text{矩形}ABCD}} = \frac{7x \times 2x}{\sqrt{17}x \times 2\sqrt{17}x} = \frac{7}{17}$ , 故选 B.

5.  $\frac{25}{4}$  【解析】作  $AM \perp PD$  于  $M$ ,  $CN \perp PA$  于  $N$ ,  $\therefore \angle AMD = \angle CNB = 90^\circ$ .  $\because AD = BC$ ,  $\angle ADC = \angle ABC$ ,  $\therefore \triangle AMD \cong \triangle CNB$  (AAS),  $\therefore AM = CN$ ,  $NB = DM$ .  $\because \angle APM = \angle CPN$ ,  $\angle PNC = \angle PMA = 90^\circ$ ,  $AM = CN$ ,  $\therefore \triangle PCN \cong \triangle PAM$  (AAS),  $\therefore PN = PM$ .  $\therefore \tan \angle ADC = \frac{AM}{MD} = \frac{4}{3}$ ,  $\therefore$  令  $AM = 4x$ ,  $MD = 3x$ ,  $\therefore CN = AM = 4x$ ,  $NB = MD = 3x$ .  $\therefore PB = 3CD = 3 \times \frac{11}{4} = \frac{33}{4}$ ,  $\therefore PN = PB + BN = \frac{33}{4} + 3x$ ,  $\therefore PM = PN = \frac{33}{4} + 3x$ .  $\therefore MC = MD - CD = 3x - \frac{11}{4}$ ,  $\therefore PC = PM + MC = 6x + \frac{11}{2}$ .  $\therefore PC^2 = PN^2 + CN^2$ ,  $\therefore \left(6x + \frac{11}{2}\right)^2 = \left(3x + \frac{33}{4}\right)^2 + (4x)^2$ ,  $\therefore 16x^2 + 24x - 55 = 0$ ,  $\therefore x = \frac{5}{4}$  或  $x = -\frac{11}{4}$  (舍).  $\therefore AD = \sqrt{AM^2 + MD^2} = 5x$ ,  $\therefore AD = 5 \times \frac{5}{4} = \frac{25}{4}$ . 故答案为  $\frac{25}{4}$ .

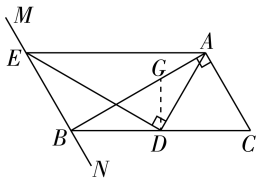
6.  $\frac{1}{8}$  【解析】过点  $F$  作  $FN \perp AC$  于点  $N$ , 过点  $D$  作  $DM \perp AC$  于点  $M$ .  $\because$  四边形  $ABCD$  是平行四边形,  $\therefore AD = BC$ ,  $AD \parallel BC$ ,  $\therefore \angle DAC = \angle ACB = 60^\circ$ ,  $\therefore \angle ADM = 30^\circ$ ,  $\therefore AM = \frac{1}{2}AD$ .  $\because \frac{AC}{BC} = \frac{5}{4}$ ,  $\therefore$  设  $AC = 5m$ , 则  $BC = 4m$ ,  $\therefore AD = BC = 4m$ ,  $\therefore AM = 2m$ ,  $DM = \sqrt{3}AM = 2\sqrt{3}m$ . 设  $EF = n$ , 则  $DF = 2n$ ,  $\therefore DE = \sqrt{3}n$ ,  $\therefore EM = \sqrt{DE^2 - DM^2} = \sqrt{3n^2 - 12m^2}$ .  $\because DE \perp GF$ ,  $\therefore \angle DEM + \angle AEG = \angle DEM + \angle FEN = 90^\circ$ .  $\because FN \perp EC$ ,  $\therefore \angle FEN + \angle EFN = 90^\circ$ ,  $\therefore \angle EFN = \angle DEM$ .  $\therefore \angle FNE = \angle EMD = 90^\circ$ ,  $\therefore \triangle EFN \sim \triangle DEM$ ,  $\therefore \frac{EN}{EF} = \frac{DM}{DE}$ ,  $\frac{FN}{FE} = \frac{ME}{DE}$ ,  $\therefore \frac{EN}{n} = \frac{2\sqrt{3}m}{\sqrt{3}n}$ ,  $\frac{FN}{n} = \frac{\sqrt{3n^2 - 12m^2}}{\sqrt{3}n}$ ,  $\therefore EN = 2m$ ,  $FN = \sqrt{n^2 - 4m^2}$ ,  $\therefore CN = CA - EN - ME - AM = 5m - 2m - \sqrt{3n^2 - 12m^2} - 2m = m - \sqrt{3n^2 - 12m^2}$ ,  $CM = CA - AM = 3m$ .  $\because FN \perp AC$ ,  $DM \perp AC$ ,  $\therefore FN \parallel DM$ ,  $\therefore \triangle CFN \sim \triangle CDM$ ,  $\therefore \frac{FN}{DM} = \frac{CN}{CM}$ ,  $\therefore \frac{\sqrt{n^2 - 4m^2}}{2\sqrt{3}m} = \frac{m - \sqrt{3n^2 - 12m^2}}{3m}$ ,  $\therefore 27n^2 = 112m^2$ ,  $\therefore CN = m - \sqrt{3n^2 - 12m^2} = \frac{1}{3}m$ .  $\therefore \triangle CFN \sim \triangle CDM$ ,  $\therefore \frac{CF}{CD} = \frac{CN}{CM} = \frac{\frac{1}{3}m}{3m} = \frac{1}{9}$ ,  $\therefore \frac{CF}{DF} = \frac{1}{8}$ .

7. (1)【证明】如图(1),过点  $D$  作  $DF \perp BC$ ,交  $AB$  于点  $F$ ,则  $\angle BDE + \angle FDE = 90^\circ$ .  
 $\because DE \perp AD, \therefore \angle FDE + \angle ADF = 90^\circ, \therefore \angle BDE = \angle ADF$ .  $\because \angle BAC = 90^\circ, \angle ABC = 45^\circ, \therefore \angle C = 45^\circ$ .  $\because MN \parallel AC, \therefore \angle EBD = 180^\circ - \angle C = 135^\circ$ .  $\because \angle FBD = 45^\circ, DF \perp BC, \therefore \angle BFD = 45^\circ, \therefore BD = DF, \angle AFD = 135^\circ, \therefore \angle EBD = \angle AFD$ . 在  $\triangle BDE$  和

$$\triangle FDA \text{ 中, } \begin{cases} \angle EBD = \angle AFD, \\ BD = DF, \\ \angle BDE = \angle ADF, \end{cases} \therefore \triangle BDE \cong \triangle FDA (ASA), \therefore AD = DE.$$



图(1)



图(2)

【解】(2)  $DE = \sqrt{3}AD$ ,理由:如图(2),过点  $D$  作  $DG \perp BC$ ,交  $AB$  于点  $G$ ,则  $\angle BDE + \angle GDE = 90^\circ$ .  $\because DE \perp AD, \therefore \angle GDE + \angle ADG = 90^\circ, \therefore \angle BDE = \angle ADG$ .  $\because \angle BAC = 90^\circ, \angle ABC = 30^\circ, \therefore \angle C = 60^\circ$ .  $\because MN \parallel AC, \therefore \angle EBD = 180^\circ - \angle C = 120^\circ$ .  $\because \angle ABC = 30^\circ, DG \perp BC, \therefore \angle BGD = 60^\circ, \therefore \angle AGD = 120^\circ, \therefore \angle EBD = \angle AGD, \therefore \triangle BDE \sim \triangle GDA, \therefore \frac{AD}{DE} = \frac{DG}{BD}$ . 在  $\text{Rt}\triangle BDG$  中,  $\frac{DG}{BD} = \tan 30^\circ = \frac{\sqrt{3}}{3}, \therefore DE = \sqrt{3}AD$ .

(3)  $AD = DE \cdot \tan \alpha$ . 如图(2),由(2)知  $\angle BDE = \angle ADG$ .  $\because AC \parallel MN, \therefore$  易得  $\angle EBD = 90^\circ + \alpha$ . 又  $\because \angle AGD = 90^\circ + \alpha, \therefore \angle EBD = \angle AGD, \therefore \triangle EBD \sim \triangle AGD, \therefore \frac{AD}{DE} = \frac{DG}{BD}$ . 在  $\text{Rt}\triangle BDG$  中,  $\frac{DG}{BD} = \tan \alpha$ , 则  $\frac{AD}{DE} = \tan \alpha, \therefore AD = DE \cdot \tan \alpha$ .

8. 【解】(1)  $D_1E = D_2F$ ,证明:  $\because C_1D_1 \parallel C_2D_2, \therefore \angle AC_1D_1 = \angle AFD_2$ . 又  $\because \angle ACB = 90^\circ, CD$  是斜边上的中线,  $\therefore DC = DA = DB$ , 即  $C_1D_1 = C_2D_2 = BD_2 = AD_1, \therefore \angle AC_1D_1 = \angle A, \therefore \angle AFD_2 = \angle A, \therefore AD_2 = D_2F$ . 同理,  $BD_1 = D_1E$ . 又  $\because AD_1 = BD_2, \therefore AD_2 = BD_1, \therefore D_1E = D_2F$ .

(2)  $\because$  在  $\text{Rt}\triangle ABC$  中,  $AC = 8, BC = 6, \therefore$  由勾股定理,得  $AB = 10$ , 即  $AD_1 = BD_2 = C_1D_1 = C_2D_2 = 5$ . 又  $\because D_2D_1 = x, \therefore D_1E = BD_1 = D_2F = AD_2 = 5 - x, \therefore C_2F = C_1E = x$ .  $\because$  在  $\triangle BC_2D_2$  中,  $C_2$  到  $BD_2$  的距离就是  $\triangle ABC$  的  $AB$  边上的高,为  $\frac{24}{5}, \therefore \triangle BC_2D_2$  的面

积为  $\frac{1}{2} \times 5 \times \frac{24}{5} = 12$ . 设  $\triangle BED_1$  的  $BD_1$  边上的高为  $h$ .  $\because C_1D_1 \parallel C_2D_2, \therefore \triangle BC_2D_2 \sim \triangle BED_1, \therefore \frac{5h}{24} = \frac{5-x}{5}, \therefore h = \frac{24(5-x)}{25}, \therefore \triangle BED_1$  的面积为  $\frac{1}{2} \times BD_1 \times h = \frac{1}{2} \times (5-x) \times$

$\frac{24(5-x)}{25} = \frac{12}{25} (5-x)^2$ .  $\because \angle C_1 + \angle C_2 = 90^\circ, \angle C_1 = \angle C_2FP, \therefore \angle FPC_2 = 90^\circ$ .

又  $\because \angle BC_2D_2 = \angle B, \therefore$  易得  $\sin \angle PC_2F = \sin B = \frac{4}{5}, \cos \angle PC_2F = \cos B = \frac{3}{5}, \therefore PC_2 = \frac{3}{5}x, PF = \frac{4}{5}x. \therefore \triangle C_2FP$  的面积为  $\frac{6}{25}x^2$ , 故重叠部分面积 =  $\triangle BC_2D_2$  的面积 -

$\triangle BED_1$  的面积 -  $\triangle C_2FP$  的面积,  $\therefore y = -\frac{18}{25}x^2 + \frac{24}{5}x. (0 \leq x \leq 5)$