

点, $\therefore AE=EC$.

又 $\because \angle AFE = \angle H = 90^\circ$, $\angle AEF = \angle CEH$,
 $\therefore \triangle AEF \cong \triangle CEH$ (AAS), $\therefore AF = CH$,
 $EF = EH$.

$\therefore \triangle AEF \sim \triangle BEA$, $\therefore \frac{AB}{AE} = \frac{AF}{EF}$.

又 $\because AB=AC=2AE$, $\therefore AF=2EF$, $\therefore CH=AF=$
 $EF+EH=FH$, $\therefore \angle CFH = 45^\circ$, $\therefore \angle AFC =$
 $\angle AFE + \angle CFH = 135^\circ$.

(3) 如图, 由 (2) 得 $\angle CFH = 45^\circ = \angle ACB$,
 $\therefore \angle FBC + \angle FCB = \angle ACF + \angle FCB$, $\angle BFC =$
 $\angle AFC = 135^\circ$, $\therefore \angle ACF = \angle FBC$, $\therefore \triangle AFC \sim$
 $\triangle CFB$, $\therefore \frac{BF}{CF} = \frac{CF}{AF} = \frac{BC}{AC}$. $\therefore$ 点 D, E 分别是

BC, AC 的中点, $\therefore DB=CD, AE=EC$, $\therefore \frac{BF}{CF} =$

$\frac{BC}{AC} = \frac{BD}{CE} = \sqrt{2}$. 又 $\because \angle FBC = \angle ACF$,

$\therefore \triangle BDF \sim \triangle CEF$, $\therefore \frac{DF}{EF} = \frac{BD}{CE} = \sqrt{2}$, $\therefore EF =$

$\frac{DF}{\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$, $\therefore AF = 2EF = \sqrt{2}$. $\therefore \angle EAF +$
 $\angle AEB = \angle ABF + \angle AEB = 90^\circ$, $\therefore \angle EAF =$
 $\angle ABF$.

又 $\because \angle AFE = \angle AFB$, $\therefore \triangle AEF \sim \triangle BAF$,

$\therefore \frac{EF}{AF} = \frac{AF}{BF}$, $\therefore \frac{\frac{\sqrt{2}}{2}}{\sqrt{2}} = \frac{\sqrt{2}}{BF}$, $\therefore BF = 2\sqrt{2}$, $\therefore \triangle ABF$

的面积为 $\frac{1}{2} \times AF \times BF = \frac{1}{2} \times \sqrt{2} \times 2\sqrt{2} = 2$.

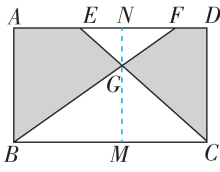
7.3 相似三角形的性质

课时 1 相似三角形对应线段的比

刷基础

1. **D** 【解析】 $\because \triangle ABC \sim \triangle A'B'C'$, AD, BE 分别是 $\triangle ABC$ 的高和中线, $A'D', B'E'$ 分别是 $\triangle A'B'C'$ 的高和中线, $\therefore \frac{AD}{A'D'} = \frac{BE}{B'E'} = \frac{AB}{A'B'}$.
 $\because AD = 4, A'D' = 3, BE = 6$, $\therefore \frac{4}{3} = \frac{6}{B'E'}$,
 $\therefore B'E' = \frac{9}{2}$. 故选 D.

2. **C** 【解析】如图, 过点 G 作 $GN \perp AD$ 于 N , 延长 NG 交 BC 于 M . $\because$ 四边形 $ABCD$ 是矩形, $\therefore AD =$



思路分析

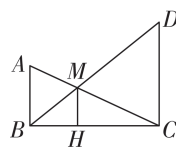
(3) 通过证明 $\triangle BDF \sim \triangle CEF$, 可得 $\frac{DF}{EF} = \frac{BD}{CE}$, 进而可求 EF . 由 $AF = 2EF$ 可求 AF , 证明 $\triangle AEF \sim \triangle BAF$, 得 $\frac{EF}{AF} = \frac{AF}{BF}$, 可求 BF , 进而由三角形面积公式可求解.

$BC, AD \parallel BC$. $\therefore EF = \frac{1}{2}AD$, $\therefore EF = \frac{1}{2}BC$.
 $\because AD \parallel BC, NG \perp AD$, $\therefore \triangle EFG \sim \triangle CBG, GM \perp$
 BC , $\therefore GN:GM = EF:BC = 1:2$. 又 $\because MN = AB =$
 $6, BC = 10$, $\therefore GN = 2, GM = 4, EF = 5$, $\therefore S_{\triangle BCG} =$
 $\frac{1}{2} \times 10 \times 4 = 20, S_{\triangle EFG} = \frac{1}{2} \times 5 \times 2 = 5$.
 $\therefore S_{\text{矩形}ABCD} = 6 \times 10 = 60$, $\therefore S_{\text{阴影}} = 60 - 20 - 5 = 35$.
 故选 C.

3. B

识图解题

如图, 已知 $AB \parallel MH \parallel DC$.



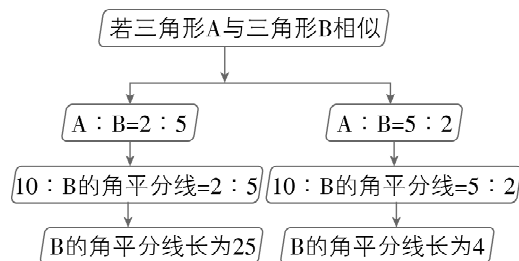
结论: $\frac{1}{AB} + \frac{1}{CD} = \frac{1}{MH}$.

【解析】 $\because AB \parallel CD$, $\therefore \triangle ABM \sim \triangle CDM$,
 $\therefore \frac{BH}{HC} = \frac{AB}{CD} = \frac{8}{12} = \frac{2}{3}$. $\because MH \parallel AB$, $\therefore \triangle MCH \sim$
 $\triangle ACB$, $\therefore \frac{MH}{AB} = \frac{CH}{BC} = \frac{3}{5}$, $\therefore \frac{MH}{8} = \frac{3}{5}$, $\therefore MH =$
 $\frac{24}{5}$ m. 故选 B.

4. 4 或 25

思路分析

相似三角形的相似比 = 对应角平分线的比.



关键点拨

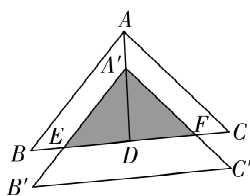
将阴影部分的面积转化为矩形与两个三角形的面积的差, 根据相似三角形的判定与性质求出两个三角形的面积是关键.

5. 2 【解析】如图所示,

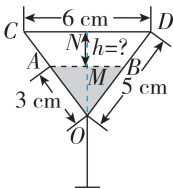
易知 $\triangle ABC \sim \triangle A'EF$,
 $A'D$ 为 $\triangle A'EF$ 的中线.

$\therefore S_{\triangle ABC} = 9, S_{\triangle A'EF} = 4$,
 $\therefore \frac{S_{\triangle ABC}}{S_{\triangle A'EF}} = \frac{9}{4}$, $\therefore \frac{AB}{A'E} =$

$\frac{AC}{A'F} = \frac{BC}{EF} = \frac{AD}{A'D} = \sqrt{\frac{9}{4}} = \frac{3}{2}$. $\therefore AD = AA' + A'D =$
 $1 + A'D$, $\therefore \frac{A'D+1}{A'D} = \frac{3}{2}$, 解得 $A'D = 2$.



6. $\frac{8}{5}$ 【解析】如图,过 O 作 $ON \perp CD$ 于 N ,交 AB 于 M .
 $\because CD \parallel AB, \therefore OM \perp AB$.
 $\because OC = OD, \therefore CN = \frac{1}{2} CD = 3 \text{ cm}, \therefore ON = \sqrt{OC^2 - CN^2} = \sqrt{5^2 - 3^2} = 4 \text{ (cm)}$.
 $\because CD \parallel AB, \therefore \triangle CDO \sim \triangle ABO, \therefore \frac{OA}{OC} = \frac{OM}{ON}$,
 $\therefore \frac{3}{5} = \frac{OM}{4}, \therefore OM = \frac{12}{5} \text{ cm}, \therefore h = 4 - \frac{12}{5} = \frac{8}{5} \text{ (cm)}$,故答案为 $\frac{8}{5}$.



关键点拨

本题考查相似三角形的实际应用,找出相似关系是解题关键.

7. (1) 【证明】 $\because AG \perp BC, AF \perp DE, \therefore \angle AFD = \angle AGC = 90^\circ$.
 $\therefore \angle DAF = \angle GAC, \therefore \angle ADE = \angle ACB$.
 $\therefore \angle EAD = \angle BAC, \therefore \triangle ADE \sim \triangle ACB$.
 (2) 【解】由(1)知 $\triangle ADE \sim \triangle ACB$.
 $\because AF \perp DE, AG \perp BC, \therefore \frac{AF}{AG} = \frac{AE}{AB}$.
 $\because AF = 4, AE = 5, AB = 12, \therefore \frac{4}{AG} = \frac{5}{12}$,
 $\therefore AG = \frac{48}{5}$.

关键点拨

根据相似三角形周长的比等于相似比,面积的比等于相似比的平方求出面积比,再分两种情况计算另一个三角形的面积即可.

课时2 相似三角形周长的比、面积的比

刷基础

1. A 【解析】设小三角形的周长为 x ,则大三角形的周长为 $x+40$. $\because$ 两个相似三角形的一组对应边的长分别是 15 和 23, $\therefore$ 相似比为 15:23, $\therefore$ 两三角形的周长比为 15:23, $\therefore \frac{x}{x+40} = \frac{15}{23}$,解得 $x=75$, $\therefore$ 小三角形的周长为 75,大三角形的周长为 $75+40=115$. 故选 A.

2. (1) 【证明】 $\because$ 四边形 $ABCD$ 为平行四边形, E 是 CD 延长线上一点, $\therefore AB \parallel CE, \angle A = \angle C$,
 $\therefore \angle ABE = \angle E, \therefore \triangle ABF \sim \triangle CEB$.

- (2) 【解】 $\because AB \parallel CE, \therefore \triangle ABF \sim \triangle DEF$,
 $\therefore \frac{C_{\triangle DEF}}{C_{\triangle ABF}} = \frac{DE}{AB}, \therefore DE = \frac{1}{2} CD = \frac{1}{2} AB, \therefore \frac{DE}{AB} = \frac{1}{2}$,
 $\therefore \frac{C_{\triangle DEF}}{C_{\triangle ABF}} = \frac{1}{2}$. 又 $\because \triangle DEF$ 的周长为 5, $\therefore \triangle ABF$ 的周长为 10.

刷有所得

相似三角形的周长比等于相似比.

易错警示

相似三角形面积的比等于相似比的平方,而不是等于相似比.

3. D 【解析】由题图可得 $S_{\triangle ABC} = \frac{1}{2} \times 3 \times 2 = 3$,
 $BC = 3, CE = 4. \therefore \triangle ABC \sim \triangle FCE, \therefore \frac{S_{\triangle ABC}}{S_{\triangle FCE}} =$

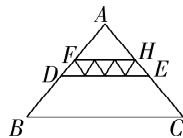
$$\left(\frac{BC}{CE}\right)^2, \text{即 } \frac{3}{S_{\triangle FCE}} = \left(\frac{3}{4}\right)^2, \therefore S_{\triangle FCE} = \frac{16}{3}. \text{ 故选 D.}$$

4. 26 【解析】如图,根据题意

$$\text{得 } \triangle AFH \sim \triangle ADE, \therefore \frac{S_{\triangle AFH}}{S_{\triangle ADE}} =$$

$$\left(\frac{FH}{DE}\right)^2 = \left(\frac{3}{4}\right)^2 = \frac{9}{16}. \text{ 设}$$

$S_{\triangle AFH} = 9x$, 则 $S_{\triangle ADE} = 16x$. 根据题意,得 $16x - 9x = 7$, 解得 $x = 1, \therefore S_{\triangle ADE} = 16, \therefore$ 四边形 $DBCE$ 的面积为 $42 - 16 = 26$. 故答案为 26.



5. $\frac{3}{4}$ 或 12 【解析】 $\because$ 两个相似三角形的周长之比是 1:2, $\therefore$ 这两个相似三角形的相似比为 1:2, $\therefore$ 它们的面积比为 1:4. 设另一个三角形的面积为 S . 若 $S > 3$, 则 $\frac{3}{S} = \frac{1}{4}$, 解得 $S = 12$; 若 $S < 3$, 则 $\frac{S}{3} = \frac{1}{4}$, 解得 $S = \frac{3}{4}$. 故答案为 $\frac{3}{4}$ 或 12.

6. 【解】(1) $\because \frac{CB}{CD} = \frac{CA}{CB} = \frac{3}{2}$, 且 $\angle C = \angle C$,

$$\therefore \triangle ACB \sim \triangle BCD, \therefore \frac{C_{\triangle ABC}}{C_{\triangle BCD}} = \frac{AC}{BC} = \frac{3}{2},$$

$$\therefore \triangle ABC \text{ 的周长为 } \frac{3}{2} \times 24 = 36 \text{ (cm)}.$$

$$(2) \because \triangle BCD \sim \triangle ACB, \therefore \frac{S_{\triangle BCD}}{S_{\triangle ABC}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9},$$

$\therefore \triangle BCD$ 与 $\triangle ABC$ 的面积比为 4:9.

7. 4:5 12 cm 【解析】 $\because$ 相似多边形面积之比等于相似比的平方, $\therefore$ 它们的相似比是

$$\sqrt{\frac{16}{25}} = \frac{4}{5} = 4:5. \therefore \text{周长之比等于相似比, 四边形 } A'B'C'D' \text{ 的周长为 } 15 \text{ cm}, \therefore \text{ 四边形 } ABCD \text{ 的}$$

$$\text{周长为 } 15 \times \frac{4}{5} = 12 \text{ (cm)}.$$

刷易错

8. 45 cm² 【解析】 $\because$ 两个相似三角形的相似比为 2:3, $\therefore$ 它们的面积之比为 4:9. 设较大三角形的面积为 $x \text{ cm}^2$. 根据题意,得 $4:9 = (x - 25):x$, 解得 $x = 45$. 故较大三角形的面积是 45 cm².

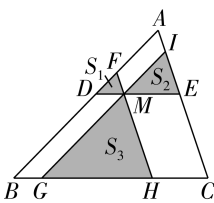
刷提升

1. B 【解析】∵ 四边形 $ABCD$ 是平行四边形，
 $\therefore CD \parallel AB, BC \parallel AD, \therefore \triangle CDE \sim \triangle BFE$ ，
 $\therefore \frac{CE}{BE} = \frac{DE}{EF} = \frac{4}{3}, \therefore \frac{EF}{FD} = \frac{3}{7}$. $\because BC \parallel AD$ ，
 $\therefore \triangle BEF \sim \triangle ADF, \therefore \triangle BEF$ 与 $\triangle ADF$ 的周长
 之比为 $3:7$ ，故选 B.

2. C 【解析】∵ 四边形 $ABCD$ 为平行四边形，
 $\therefore AB \parallel DC, AD \parallel BC, AD = BC, \therefore \angle BAE = \angle E$ ，
 $\triangle BAF \sim \triangle DEF. \therefore AG$ 平分 $\angle BAD, \therefore \angle BAE =$
 $\angle DAE, \therefore \angle DAE = \angle E, \therefore AD = DE$. 同理可得
 $AB = BG. \therefore \triangle BAF \sim \triangle DEF, \therefore \frac{BF}{DF} = \frac{AB}{ED} = \frac{AB}{AD} =$
 $\frac{2}{3}, \therefore \frac{S_{\triangle ABF}}{S_{\triangle ADF}} = \frac{BF}{DF} = \frac{2}{3}, \therefore S_{\triangle ABF} = \frac{2}{3} S_1. \therefore AD \parallel$
 $BC, \therefore \triangle ADF \sim \triangle GBF, \therefore \frac{AD}{GB} = \frac{DF}{BF} = \frac{3}{2}$ ，
 $\therefore \frac{S_{\triangle ADF}}{S_{\triangle GBF}} = \left(\frac{AD}{BG}\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}, \therefore S_{\triangle GBF} = \frac{4}{9} S_1$.
 设 $AD = BC = 3a$ ，则 $AB = GB = 2a, \therefore CG = a$.
 $\because AB \parallel DC, \therefore \triangle ABG \sim \triangle ECG, \therefore \frac{S_{\triangle ABG}}{S_{\triangle ECG}} =$
 $\left(\frac{BG}{CG}\right)^2 = \left(\frac{2a}{a}\right)^2 = 4, \therefore S_{\triangle ABG} = 4S_2. \therefore S_{\triangle ABF} +$
 $S_{\triangle GBF} = S_{\triangle ABG}, \therefore \frac{2}{3} S_1 + \frac{4}{9} S_1 = 4S_2$ ，即 $\frac{5}{9} S_1 =$
 $2S_2, \therefore \frac{S_2}{S_1} = \frac{5}{18}$ ，故选 C.

3. B 【解析】∵ $DE \parallel BC, \therefore \triangle ADE \sim \triangle ABC$ ，
 $\angle ADE = \angle B, \angle EDF = \angle DFB$. 又由折叠知
 $\angle ADE = \angle EDF, \therefore \angle B = \angle DFB, \therefore DB = DF$.
 $\therefore AD = DM = 5 \text{ cm}, BD = DF = 3 \text{ cm}, \therefore FM = 2 \text{ cm}$.
 $\therefore \frac{S_{\triangle ADE}}{S_{\triangle ABC}} = \left(\frac{AD}{AB}\right)^2$ ，即 $\frac{S_{\triangle ADE}}{32} = \left(\frac{5}{5+3}\right)^2, \therefore S_{\triangle ADE} =$
 $12.5 \text{ cm}^2, \therefore S_{\triangle MDE} = S_{\triangle ADE} = 12.5 \text{ cm}^2$ ，同理可
 得 $S_{\triangle FMG} = 2 \text{ cm}^2, \therefore$ 四边形 $DEGF$ 的面积为
 10.5 cm^2 . 故选 B.

4. 49 【解析】如图所示，由
 题意可得 $DE \parallel BC, FH \parallel$
 $AC, GI \parallel AB, \therefore \triangle DFM \sim$
 $\triangle BFH, \triangle GMH \sim \triangle BFH$ ，
 $\triangle DMF \sim \triangle DEA, \triangle MEI \sim$
 $\triangle DEA, \therefore \triangle DFM \sim \triangle GMH \sim \triangle MIE. \therefore$ 三个阴
 影小三角形的面积 S_1, S_2, S_3 分别是 1, 4 和 16，
 $\therefore DM:ME:GH = 1:2:4. \therefore DE \parallel BC, FH \parallel AC$ ，

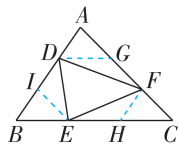


思路分析

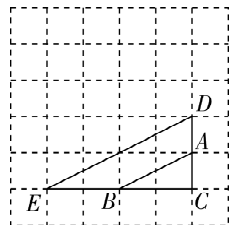
取 AF 的中点
 G, CE 的中点
 H, BD 的中点
 I ，连接 DG ，
 FH, EI ，易证
 $\triangle DAG \sim \triangle BAC$ ，
 即可推出
 $\frac{S_{\triangle DAG}}{S_{\triangle BAC}} = \frac{1}{9}$ ，根
 据等高三角形
 面积之比等于
 底边长度之
 比，可得 $S_{\triangle DAF} =$
 $2S_{\triangle DAG} = \frac{2}{9} S_{\triangle BAC}$ ，
 同理推出
 $S_{\triangle BDE} = \frac{2}{9} S_{\triangle BAC}$ ，
 $S_{\triangle CEF} = \frac{2}{9} S_{\triangle BAC}$ ，
 即可求解.

$GI \parallel AB, \therefore$ 四边形 $BDMG$ 与四边形 $CEMH$ 均
 为平行四边形， $\therefore DM = BG, EM = CH$. 设 $DM =$
 x ，则 $BC = BG + GH + CH = x + 4x + 2x = 7x, \therefore BC:$
 $DM = 7:1$. 易得 $\triangle FDM \sim \triangle ABC, \therefore S_{\triangle ABC}:$
 $S_{\triangle FDM} = 49:1, \therefore S_{\triangle ABC} = 1 \times 49 = 49$ ，故答案
 为 49.

5. $\frac{1}{3}$ 【解析】已知 D, E, F 分别
 是 $\triangle ABC$ 三边的三等分点，且
 $\frac{AD}{AB} = \frac{BE}{BC} = \frac{CF}{AC} = \frac{1}{3}$. 如图，取 AF
 的中点 G, CE 的中点 H, BD 的中点 I ，连接
 $DG, FH, EI, \therefore AG = \frac{1}{2} AF = \frac{1}{3} AC, BI = \frac{1}{2} BD =$
 $\frac{1}{3} BA, CH = \frac{1}{2} CE = \frac{1}{3} BC, \therefore \frac{AD}{AB} = \frac{AG}{AC} = \frac{1}{3}$.
 又 $\because \angle DAG = \angle BAC, \therefore \triangle DAG \sim \triangle BAC$ ，
 $\therefore \frac{S_{\triangle DAG}}{S_{\triangle BAC}} = \left(\frac{AD}{AB}\right)^2 = \frac{1}{9}$ ，即 $S_{\triangle DAG} = \frac{1}{9} S_{\triangle BAC}$.
 $\therefore AG = \frac{1}{2} AF, \therefore S_{\triangle DAF} = 2S_{\triangle DAG} = \frac{2}{9} S_{\triangle BAC}$. 同理
 可证得 $S_{\triangle BDE} = \frac{2}{9} S_{\triangle BAC}, S_{\triangle CEF} = \frac{2}{9} S_{\triangle BAC}$ ，
 $\therefore S_{\triangle DEF} = S_{\triangle BAC} - S_{\triangle ADF} - S_{\triangle BDE} - S_{\triangle CEF} =$
 $\left(1 - 3 \times \frac{2}{9}\right) S_{\triangle BAC} = \frac{1}{3} S_{\triangle BAC}, \therefore \triangle DEF$ 与 $\triangle ABC$
 的面积的比值为 $\frac{1}{3}$. 故答案为 $\frac{1}{3}$.

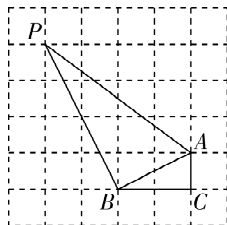


6. 【解】(1) 如图(1)所示， $\triangle CDE$ 即为所求作
 (答案不唯一).



图(1)

(2) 如图(2)所示， $\triangle ABP$ 即为所求作 (答案
 不唯一).



图(2)

关键点拨

由相似三角形
 的面积比可以
 得到相似比，
 根据相似三角
 形的相似比也
 可以得到面积
 比.

7. (1) 【证明】 $\because$ 四边形 $ABCD$ 是正方形,
 $\therefore \angle D = \angle A = 90^\circ$.

$\because HM \perp MN, \therefore \angle HMN = 90^\circ, \therefore \angle DMN + \angle AMH = \angle AHM + \angle AMH = 90^\circ, \therefore \angle AHM = \angle DMN, \therefore \triangle AHM \sim \triangle DMN$.

【解】(2) $\because$ 点 M 是 AD 的中点, 正方形 $ABCD$ 的边长为 4, $\therefore MD = AM = 2$. 设 $HM = HB = x$, 则 $AH = 4 - x$.

在 $\text{Rt} \triangle AHM$ 中, $AH^2 + AM^2 = HM^2, \therefore (4-x)^2 + 2^2 = x^2$, 解得 $x = 2.5, \therefore AH = 1.5$.

$\because \triangle AHM \sim \triangle DMN, \therefore \frac{AM}{DN} = \frac{MH}{MN} = \frac{AH}{MD}$,

即 $\frac{2}{DN} = \frac{2.5}{MN} = \frac{1.5}{2}, \therefore DN = \frac{8}{3}, MN = \frac{10}{3}$,

$\therefore \triangle DMN$ 的周长为 $DN + MN + DM = \frac{8}{3} + \frac{10}{3} + 2 = 8$.

(3) $\triangle DMN$ 的周长是一个定值, 始终为 8. 理由如下:

设 $HM = HB = x$, 则 $AH = 4 - x$. 在 $\text{Rt} \triangle AHM$ 中, $AH^2 + AM^2 = HM^2, \therefore (4-x)^2 + AM^2 = x^2$, 即 $AM = \sqrt{8x-16}, \therefore DM = 4 - AM = 4 - \sqrt{8x-16}$.

$\because \triangle AHM \sim \triangle DMN$, 且相似比为 $\frac{AH}{MD} =$

$\frac{4-x}{4-\sqrt{8x-16}}, \therefore C_{\triangle DMN} = C_{\triangle AHM} \cdot \frac{4-\sqrt{8x-16}}{4-x} =$

$(4-x+x+\sqrt{8x-16}) \cdot \frac{4-\sqrt{8x-16}}{4-x} = 8$.

$\therefore$ 在动点 H 逐渐向点 A 运动的过程中, $\triangle DMN$ 的周长是一个定值, 始终为 8.

刷素养

8. 【解】(1) ① $\because DE \parallel BC, \therefore \triangle ADE \sim \triangle ACB$.

$\because DE$ 平分 $\triangle ABC$ 的面积, $\therefore \frac{S_{\triangle ADE}}{S_{\triangle ACB}} = \frac{1}{2}$,

$\therefore \frac{AD}{AC} = \frac{1}{\sqrt{2}}$, 即 $\frac{AD}{3} = \frac{1}{\sqrt{2}}$, 解得 $AD = \frac{3\sqrt{2}}{2}$.

② 在 $\triangle ABC$ 中, $\angle C = 90^\circ, AC = 3, BC = 4, \therefore AB = \sqrt{AC^2 + BC^2} = \sqrt{3^2 + 4^2} = 5, \therefore \triangle ABC$ 的周长为 $3+4+5=12$.

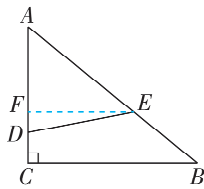
$\because DE$ 平分 $\triangle ABC$ 的周长, $\therefore AD + AE = 6$, 即 $AE = 6 - AD. \because DE \parallel BC, \therefore \triangle ADE \sim \triangle ACB$,

$\therefore \frac{AD}{AC} = \frac{AE}{AB}$, 即 $\frac{AD}{3} = \frac{6-AD}{5}$, 解得 $AD = \frac{9}{4}$.

思路分析

(2) 过点 E 作 $EF \perp AC$ 于 F , 设 $AD = x$, 根据相似三角形的性质用含 x 的式子表示出 EF , 再根据三角形的面积公式列方程求解即可, 注意对求出的 x 值进行取舍.

(2) 存在. 如图, 过点 E 作 $EF \perp AC$ 于 F . 设 DE 将 $\triangle ABC$ 的周长平分, 则 $AD + AE = 6$.



设 $AD = x$, 则 $AE = 6 - x. \because EF \parallel BC, \therefore \triangle AEF \sim \triangle ABC, \therefore \frac{EF}{BC} = \frac{AE}{AB}$, 即 $\frac{EF}{4} = \frac{6-x}{5}$, 解得 $EF = \frac{24-4x}{5}, \therefore S_{\triangle ADE} = \frac{1}{2} AD \cdot EF = \frac{1}{2} x \cdot \frac{24-4x}{5} = -\frac{2}{5}x^2 + \frac{12}{5}x$.

当 DE 将 $\triangle ABC$ 的面积平时, $-\frac{2}{5}x^2 + \frac{12}{5}x =$

$\frac{1}{2} \times 3 \times 4 \times \frac{1}{2}$, 解得 $x_1 = \frac{6+\sqrt{6}}{2}, x_2 = \frac{6-\sqrt{6}}{2}$.

$\because 0 < x < 3, \therefore x = \frac{6-\sqrt{6}}{2}$, 即当 $AD = \frac{6-\sqrt{6}}{2}$ 时, DE

将 $\triangle ABC$ 的周长和面积同时平分.

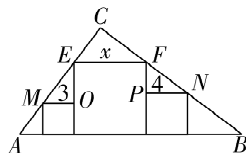


微专题

思路分析

根据已知条件可以推出 $\triangle CEF \sim \triangle OME \sim \triangle PFN$, 然后利用相似三角形对应边的比相等求解.

1. B 【解析】如图, 由题意得 $OM \parallel AB \parallel PN \parallel EF, EO \parallel FP, \angle C = \angle EOM = \angle NPF = 90^\circ$, 则 $\angle CEF = \angle CMO, \angle CFE = \angle CNP, \therefore \triangle CEF \sim \triangle OME \sim \triangle PFN, \therefore OE : PN = OM : PF. \because EF = x, MO = 3, PN = 4, \therefore$ 易得 $OE = x - 3, PF = x - 4, \therefore (x-3) : 4 = 3 : (x-4), \therefore (x-3)(x-4) = 12$, 即 $x^2 - 4x - 3x + 12 = 12, \therefore x = 0$ (不符合题意, 舍去) 或 7. 故选 B.



关键点拨

利用相似三角形的性质求出 EF 的长是解题的关键.

2. 1 : 3 【解析】 $\because$ 四边形 $EFGH$ 和四边形 $HGNM$ 均为正方形, $\therefore EF = EH = HG = HM, EM \parallel BC, \therefore \triangle AEM \sim \triangle ABC, \therefore \frac{AP}{AD} = \frac{EM}{BC}$,

$\therefore \frac{5-EF}{5} = \frac{2EF}{10}, \therefore EF = \frac{5}{2}, \therefore EM = 5. \because \triangle AEM \sim$

$\triangle ABC, \therefore \frac{S_{\triangle AEM}}{S_{\triangle ABC}} = \left(\frac{EM}{BC}\right)^2 = \frac{1}{4}, \therefore S_{\text{四边形}BCME} =$

$S_{\triangle ABC} - S_{\triangle AEM} = 3S_{\triangle AEM}, \therefore \triangle AEM$ 与四边形 $BCME$ 的面积比为 1 : 3, 故答案为 1 : 3.

重难专题2 相似三角形与其他

知识的综合

刷难关

1.【解】(1)如图,点M即为所求.

(2)如图, $\because \triangle AMO \sim \triangle AOB$,

$$\therefore AO:AB = AM:AO, \therefore OA^2 = AM \cdot AB.$$

$$\because A(6,0), \therefore OA=6.$$

$$\because AB=4AM, \therefore AM \times 4AM = 36, \therefore AM=3,$$

$$\therefore AB=12, \therefore OB = \sqrt{AB^2 - AO^2} = \sqrt{12^2 - 6^2} = 6\sqrt{3},$$

$$\therefore B(0,6\sqrt{3}).$$

$$\because AC=BC, \therefore C(3,3\sqrt{3}), \therefore \text{易得直线 } OC \text{ 的表达式为 } y = \sqrt{3}x.$$

$$\because \angle AMO = \angle AOB = 90^\circ,$$

$$\therefore \frac{1}{2} \cdot OA \cdot OB = \frac{1}{2} \cdot OM \cdot AB, \therefore OM =$$

$$\frac{6 \times 6\sqrt{3}}{12} = 3\sqrt{3}, \therefore \text{易得点 } M \text{ 坐标为 } \left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right).$$

$$\because MO=MF, \therefore \text{易得 } F(9,0), \therefore \text{易得直线 } MF \text{ 的表达式为 } y = -\frac{\sqrt{3}}{3}x + 3\sqrt{3}.$$

$$\text{由 } \begin{cases} y = -\frac{\sqrt{3}}{3}x + 3\sqrt{3}, \\ y = \sqrt{3}x, \end{cases} \text{ 解得 } \begin{cases} x = \frac{9}{4}, \\ y = \frac{9\sqrt{3}}{4}, \end{cases}$$

$$\therefore D\left(\frac{9}{4}, \frac{9\sqrt{3}}{4}\right).$$

2. (1)【证明】 $\because$ 四边形ABCD是菱形, $\therefore AB \parallel CD, \angle A = \angle BCD, \therefore \angle AFD = \angle FDG$.

由对称性可 $\angle DFG = \angle BCD, \therefore \angle A = \angle DFG,$

$$\therefore \triangle DFG \sim \triangle FAD.$$

(2)【解】①如图,连接AC,与BD相交于O.

$\because$ 四边形ABCD是菱形, $\therefore \angle AOB = 90^\circ,$

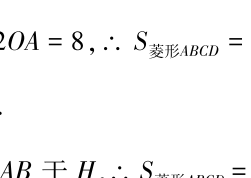
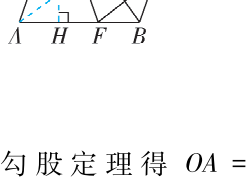
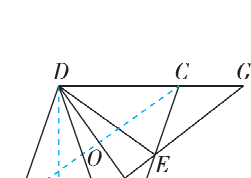
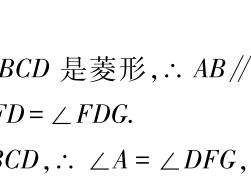
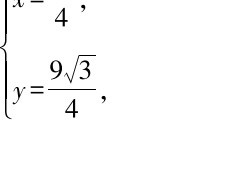
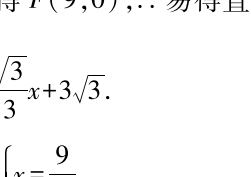
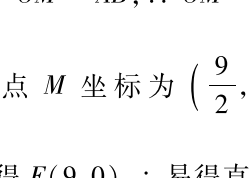
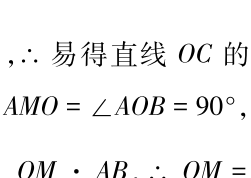
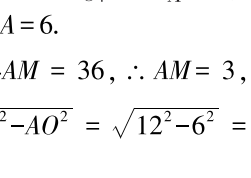
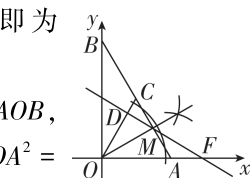
$$OB = \frac{1}{2}BD = 3, AC = 2OA.$$

在Rt $\triangle AOB$ 中,根据勾股定理得 $OA =$

$$\sqrt{AB^2 - OB^2} = 4, \therefore AC = 2OA = 8, \therefore S_{\text{菱形}ABCD} =$$

$$\frac{1}{2}AC \cdot BD = \frac{1}{2} \times 8 \times 6 = 24.$$

②如图,过点D作 $DH \perp AB$ 于H, $\therefore S_{\text{菱形}ABCD} =$



思路分析

(2) ①连接AC,与BD相交于O,先利用勾股定理求出OA,进而得出AC,即可计算面积;

②过点D作 $DH \perp AB$,先利用菱形的面积求出DH,再用勾股定理求出AH,进而得出AF,最后借助(1)的结论得出 $\frac{DG}{DF} = \frac{DF}{AF}$,求出DG,即可求出CG.

思路分析

(2) 过点C作 $CH \perp PQ$ 于H,过点B作 $BT \perp PQ$ 于T.

由 $AP = \frac{1}{4}AC$,可设 $AP = CQ = a$,则 $PC = 3a$,进而求出CH,BT,利用相似三角形对应边成比例求解即可.

$$AB \cdot DH = 5DH.$$

$$\text{由①知 } S_{\text{菱形}ABCD} = 24, \therefore 5DH = 24, \therefore DH = \frac{24}{5}.$$

在Rt $\triangle DAH$ 中, $AH = \sqrt{AD^2 - DH^2} =$

$$\sqrt{5^2 - \left(\frac{24}{5}\right)^2} = \frac{7}{5}.$$

由对称知 $DF = CD = 5, \therefore AD = DF.$

$$\because DH \perp AF, \therefore AF = 2AH = \frac{14}{5}.$$

$$\text{由(1)知 } \triangle DFG \sim \triangle FAD, \therefore \frac{DG}{DF} = \frac{DF}{AF},$$

$$\therefore \frac{DG}{5} = \frac{5}{\frac{14}{5}}, \therefore DG = \frac{125}{14}, \therefore CG = DG - CD = \frac{55}{14}.$$

3. (1)【证明】 $\because$ 线段BP绕点B顺时针旋转 90° 得到线段BQ, $\therefore BP = BQ, \angle PBQ = 90^\circ.$

$\because$ 四边形ABCD是正方形, $\therefore BA = BC, \angle ABC = 90^\circ,$

$\therefore \angle ABC = \angle PBQ, \therefore \angle ABC - \angle PBC = \angle PBQ - \angle PBC,$

即 $\angle ABP = \angle CBQ.$ 在 $\triangle BAP$

和 $\triangle BCQ$ 中, $\because \begin{cases} BA = BC, \\ \angle ABP = \angle CBQ, \\ BP = BQ, \end{cases} \therefore \triangle BAP \cong \triangle BCQ$ (SAS), $\therefore CQ = AP.$

(2)【解】如图,过点C作 $CH \perp PQ$ 于H,过点B作 $BT \perp PQ$ 于T.

$$\because AP = \frac{1}{4}AC, \therefore \text{设 } AP = CQ = a, \text{ 则 } PC = 3a,$$

$\because$ 四边形ABCD是正方形, $\therefore \angle BAC = \angle ACB = 45^\circ.$

$\because \triangle ABP \cong \triangle BCQ, \therefore \angle BCQ = \angle BAP = 45^\circ, \therefore \angle PCQ = 90^\circ, \therefore PQ =$

$$\sqrt{PC^2 + CQ^2} = \sqrt{(3a)^2 + a^2} = \sqrt{10}a.$$

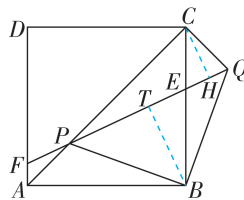
$$\because CH \perp PQ, \therefore \frac{1}{2}PC \cdot CQ = \frac{1}{2}PQ \cdot CH,$$

$$\therefore CH = \frac{PC \cdot CQ}{PQ} = \frac{3\sqrt{10}}{10}a.$$

$$\because BP = BQ, BT \perp PQ, \therefore PT = TQ. \therefore \angle PBQ =$$

$$90^\circ, \therefore BT = \frac{1}{2}PQ = \frac{\sqrt{10}}{2}a. \therefore \text{易证 } \triangle CHE \sim$$

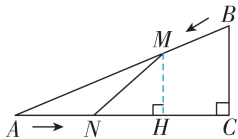
$$\triangle BTE, \therefore \frac{CE}{EB} = \frac{CH}{BT} = \frac{\frac{3\sqrt{10}}{10}a}{\frac{\sqrt{10}}{2}a} = \frac{3}{5}, \therefore \frac{CE}{CB} = \frac{3}{8}.$$



4. 【解】(1) 在 $\text{Rt} \triangle ABC$ 中, $\angle C = 90^\circ$, $BC = 5$ cm, $AC = 12$ cm,

$$\therefore AB = \sqrt{AC^2 + BC^2} = \sqrt{12^2 + 5^2} = 13 \text{ (cm)}.$$

(2) 如图, 作 $MH \perp AC$ 于点 H , 则 $MH \parallel BC$.



由题意得 $BM = 2t$ cm, $AN = t$ cm, 则 $AM = (13 - 2t)$ cm. $\because MH \parallel BC, \therefore \triangle AMH \sim \triangle ABC$,

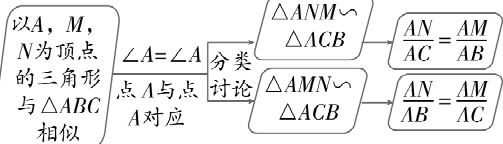
$$\therefore \frac{MH}{BC} = \frac{AM}{AB}, \text{ 即 } \frac{MH}{5} = \frac{13-2t}{13}, \therefore MH = \frac{65-10t}{13} \text{ cm}.$$

$$\text{由题意得 } \frac{1}{2} \times t \times \frac{65-10t}{13} = \frac{1}{2} \times 5 \times 12 \times \frac{3}{26}, \text{ 解得 } t_1 = 2, t_2 = \frac{9}{2}, \therefore t \text{ 为 } 2 \text{ 或 } \frac{9}{2} \text{ 时,}$$

$\triangle AMN$ 的面积为 $\triangle ABC$ 面积的 $\frac{3}{26}$.

(3)

思路分析 | 相似三角形的分类讨论



存在. $\because \angle A = \angle A, \therefore$ 当 $\triangle ANM \sim \triangle ACB$ 时,

$$\frac{AN}{AC} = \frac{AM}{AB}, \therefore \frac{t}{12} = \frac{13-2t}{13}, \text{ 解得 } t = \frac{156}{37};$$

$$\text{当 } \triangle AMN \sim \triangle ACB \text{ 时, } \frac{AN}{AB} = \frac{AM}{AC}, \therefore \frac{t}{13} = \frac{13-2t}{12},$$

$$\text{解得 } t = \frac{169}{38}.$$

综上所述, 存在 t 值使得以 A, M, N 为顶点的

三角形与 $\triangle ABC$ 相似, t 的值为 $\frac{156}{37}$ 或 $\frac{169}{38}$.

7.4 相似三角形的应用

刷基础

1. C 【解析】 $\because DF = 20$ cm $= 0.2$ m, $\therefore CF = 1 -$

$0.2 = 0.8$ (m). 由题意得 $AD \parallel BE, \therefore \triangle ADF \sim$

$$\triangle ECF, \therefore \frac{AD}{EC} = \frac{DF}{CF}, \therefore \frac{1}{CE} = \frac{0.2}{0.8}, \text{ 解得 } CE = 4,$$

$$\therefore BE = BC + CE = 1 + 4 = 5 \text{ (m)}. \text{ 故选 C.}$$

2. 10 【解析】 $\because DE \parallel BC, BC = 8$ 米, $DE = 14$ 米,

$$\therefore \triangle ABC \sim \triangle ADE, \therefore \frac{AC}{AE} = \frac{BC}{DE} = \frac{8}{14} = \frac{4}{7},$$

易错警示

注意(3)需要分类讨论, 不要漏解.

思路分析

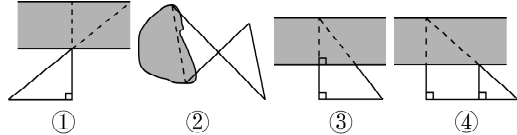
证明 $\triangle CDF \sim \triangle FDE$, 可得 $\frac{CD}{FD} = \frac{DF}{DE}$. 根据在 A 时测得某树的影长为 3 m, B 时测得该树的影长为 2 m, 可知 $CD = 2$ m, $DE = 3$ m, 代入值求解即可得到树的高度.

$$\therefore \frac{AC}{EC} = \frac{4}{3}. \because AF \perp BC, EG \perp BC, \therefore AF \parallel EG,$$

$$\therefore \triangle ACF \sim \triangle ECG, \therefore \frac{AF}{EG} = \frac{AC}{EC}, \text{ 即 } \frac{AF}{7.5} = \frac{4}{3}, \text{ 解}$$

得 $AF = 10, \therefore$ 河流的宽度 AF 为 10 米.

刷有所得 | 测量河流、池塘宽度的其他常见图示



3. A 【解析】如图所示, A 时 B 时
 $\because CF \perp EF, \therefore \angle CFD + \angle EFD = 90^\circ. \because FD \perp CE,$
 $\therefore \angle FCD + \angle CFD = 90^\circ, \therefore \angle EFD = \angle FCD. \because \angle FDC = \angle FDE = 90^\circ,$
 $\therefore \triangle CDF \sim \triangle FDE, \therefore \frac{CD}{DF} = \frac{DF}{DE}. \because CD = 2, DE = 3,$
 $\therefore \frac{2}{DF} = \frac{DF}{3}, \text{ 解得 } DF = \sqrt{6} \text{ (负值已舍去)}, \therefore$ 树的高度为 $\sqrt{6}$ m. 故选 A.

4. 1.8 【解析】由题意, 得 $CE \parallel BF, AB \parallel CD,$

$$\therefore \triangle COE \sim \triangle BOF, \triangle AOB \sim \triangle DOC, \therefore \frac{CE}{BF} =$$

$$\frac{CO}{OB}, \frac{CD}{AB} = \frac{CO}{OB}, \therefore \frac{CE}{BF} = \frac{CD}{AB}. \because BF = 2 \text{ m}, CE =$$

$$1 \text{ m}, CD = 0.9 \text{ m}, \therefore \frac{1}{2} = \frac{0.9}{AB}, \therefore AB = 1.8 \text{ m}, \text{ 即}$$

物体的高度 AB 为 1.8 m. 故答案为 1.8.

5. 【解】根据题意, 得 $\angle MNC = \angle ABC = \angle EFD =$

$$90^\circ, \angle ACB = \angle MCN, BC = 3, AB = 1.6, EF = 2.4, DF = 6, CF = 28, \therefore \triangle MNC \sim \triangle ABC,$$

$$\therefore \frac{MN}{AB} = \frac{NC}{BC}. \text{ 设 } MN = x \text{ 米}, \therefore \frac{x}{1.6} = \frac{CN}{3}, \therefore CN =$$

$$\frac{15x}{8}. \because \angle EFD = \angle MND, \angle D = \angle D, \therefore \triangle MND \sim$$

$$\triangle EFD, \therefore \frac{MN}{EF} = \frac{ND}{FD}. \therefore ND = NC + CF + DF =$$

$$\frac{15x}{8} + 28 + 6 = \frac{15x}{8} + 34, \therefore \frac{x}{2.4} = \frac{\frac{15x}{8} + 34}{6}, \text{ 解得 } x =$$

54.4, 故大雁塔的高度 MN 为 54.4 米.

刷提升

1. A 【解析】示意图如图, 过 B 作 $BE \perp OP$ 于

E , 过 D 作 $DF \perp OP$ 交 AB 于 M , 交 OP 于 F ,

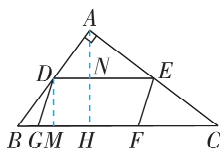
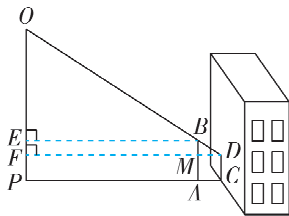
$\therefore$ 四边形 $ACDM$ 和四边形 $ABEP$ 为矩形,

$\therefore MD = AC = 0.9$, O
 $AM = CD = 1$, $BE =$
 $AP = 6.6$, $EP = AB =$
 1.6 , $\therefore BM = AB -$
 $AM = 0.6$. $\therefore AB \parallel$
 OP , $\therefore \angle DBM = \angle O$, $\angle DMB = \angle OFD = 90^\circ =$
 $\angle BEO$, $\therefore \triangle BDM \sim \triangle OBE$, $\therefore \frac{BM}{OE} = \frac{DM}{BE}$, 即
 $\frac{0.6}{OE} = \frac{0.9}{6.6}$, 解得 $OE = 4.4$, $\therefore OP = OE + EP = 6$,
 即路灯 OP 的高为 6 m. 故选 A.

2. C 【解析】由题意可得 $AD \parallel EB$, $\therefore \angle CFD =$
 $\angle AFB = \angle CBE$, $\triangle CDF \sim \triangle CEB$. $\therefore \angle ABF =$
 $\angle CEB = 90^\circ$, $\angle AFB = \angle CBE$, $\therefore \triangle CBE \sim$
 $\triangle AFB$, $\therefore \frac{BE}{FB} = \frac{BC}{AF} = \frac{EC}{AB}$. $\therefore \angle CEB = 90^\circ$, $BC =$
 2.5 m, $BE = 1.5$ m, $AB = 1.2$ m, $\therefore EC =$
 $\sqrt{2.5^2 - 1.5^2} = 2$ (m), 则 $\frac{1.5}{FB} = \frac{2.5}{AF} = \frac{2}{1.2}$,
 $\therefore FB = 0.9$ m, $AF = 1.5$ m. $\therefore \triangle CDF \sim \triangle CEB$,
 $\therefore \frac{DF}{BE} = \frac{CF}{CB}$, 即 $\frac{DF}{1.5} = \frac{2.5 - 0.9}{2.5}$, $\therefore DF = 0.96$ m,
 $\therefore AD = AF + DF = 1.5 + 0.96 = 2.46$ (m). 故选 C.

3. 12 cm^2 【解析】 $\because \angle BAC =$
 90° , $AB = 6$ cm, $AC = 8$ cm,
 $\therefore BC = \sqrt{AB^2 + AC^2} = 10$ cm.
 如图, 分别过 A, D 作 BC
 的垂线, 垂足分别为 H, M , AH 交 DE 于 N .
 $\therefore S_{\triangle ABC} = \frac{1}{2} AC \cdot AB = \frac{1}{2} BC \cdot AH$, $\therefore AH =$
 $\frac{AC \cdot AB}{BC} = \frac{24}{5}$ cm. $\therefore$ 四边形 $DEFG$ 是平行四边
 形, $AH \perp BC$, $\therefore DE \parallel BC$, $\therefore \triangle ADE \sim \triangle ABC$,
 $AH \perp DE$, $\therefore \frac{AN}{AH} = \frac{DE}{BC} = \frac{5}{10} = \frac{1}{2}$, $\therefore AN = \frac{1}{2} AH =$
 $\frac{12}{5}$ cm, $\therefore DM = NH = AH - AN = \frac{12}{5}$ cm,
 $\therefore S_{\square DEFG} = DE \cdot DM = 5 \times \frac{12}{5} = 12$ (cm²), 故答案
 为 12 cm^2 .

4. 12 【解析】由题意得 $AB \parallel CD$, $AE = OB = 6$.
 $\therefore AE \parallel OF$, $\therefore \angle CAE = \angle COF$, $\angle CEA =$
 $\angle CFO$, $\therefore \triangle CAE \sim \triangle COF$, $\therefore \frac{AE}{OF} = \frac{CA}{CO} = \frac{6}{10} =$



思路分析

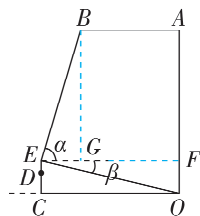
过 B 作 $BE \perp$
 OP 于 E , 过 D
 作 $DF \perp OP$ 交
 AB 于 M , 交
 OP 于 F , 则四
 边形 $ACDM$ 和
 四边形 $ABEP$
 为矩形, 因此
 可求出 $BM =$
 $AB - AM = 0.6$,
 然后证明
 $\triangle BDM \sim \triangle OBE$,
 得 $\frac{BM}{OE} = \frac{DM}{BE}$,
 求出 OE 的
 长, 最后根据
 $OP = OE + EP$
 求解即可.

思路分析

先得出 $\triangle CAE \sim$
 $\triangle COF$, 进而
 得到 $\frac{OA}{OC} = \frac{2}{5}$,
 再得出 $\triangle OAB \sim$
 $\triangle OCD$, 得到
 $\frac{OA}{OC} = \frac{AB}{CD}$, 即可
 得解.

$\frac{3}{5}$, $\therefore \frac{OA}{OC} = \frac{2}{5}$. $\therefore AB \parallel CD$, $\therefore \angle OAB = \angle OCD$,
 $\angle OBA = \angle ODC$, $\therefore \triangle OAB \sim \triangle OCD$, $\therefore \frac{OA}{OC} =$
 $\frac{AB}{CD}$, $\therefore \frac{2}{5} = \frac{4.8}{CD}$, 解得 $CD = 12$, $\therefore$ 像 CD 的高为
 12 cm, 故答案为 12.

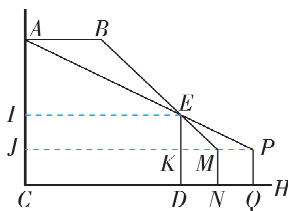
5. 8 【解析】如图, 过点 E 作
 $EF \perp AO$, 垂足为 F , 过点 B
 作 $BG \perp EF$, 垂足为 G ,
 $\therefore \angle BGE = \angle EFO = 90^\circ$,
 $\therefore \angle EBG + \angle BEG = 90^\circ$.
 $\therefore \angle BEG + \angle FEO = 90^\circ$,



$\therefore \angle EBG = \angle FEO$, $\therefore \triangle BEG \sim \triangle EFO$, $\therefore \frac{BG}{EF} =$
 $\frac{EG}{OF}$. 由题意得 $OF = CE = CD + DE = 1.2 + 0.8 =$
 2 (m), $AB = FG = 6$ m, $AF = BG$, $EF = CO$.
 $\therefore AO = 10$ m, $\therefore AF = BG = OA - OF = 10 - 2 =$
 8 (m), $\therefore \frac{8}{EG + 6} = \frac{EG}{2}$, 解得 $EG = 2$ (负值已舍
 去), $\therefore OC = EF = EG + FG = 2 + 6 = 8$ (m), $\therefore$ 裁
 判与立柱之间的水平距离 OC 为 8 m, 故答案
 为 8.

刷素养

6. 【解】如图, 过 E 作 $EI \perp AC$ 于 I , 连接 PM 并延
 长交 AC 于 J , 交 ED 于 K , 则易得 $IE = JK =$
 $CD = 8$, $KM = PM = DN = NQ = 2$, $AB \parallel IE \parallel PJ \parallel$
 CH , $\therefore \angle AEI = \angle EPK$.



$\therefore \angle AIE = \angle EKP = 90^\circ$, $\therefore \triangle AEI \sim \triangle EPK$,
 $\therefore \frac{AE}{EP} = \frac{IE}{KP} = \frac{8}{2+2} = 2$. $\therefore AB \parallel MP$, $\therefore \triangle ABE \sim$
 $\triangle PME$, $\therefore \frac{AB}{PM} = \frac{AE}{PE}$, 即 $\frac{AB}{2} = 2$, $\therefore AB = 4$.

答: 遮雨玻璃的水平宽度 AB 为 4 米.

教材素材拓展

刷拓展

1. 【解】(1) $\because EF, ED$ 为 $\triangle ABC$ 的中位线,
 $\therefore ED \parallel AB, EF \parallel BC, EF = \frac{1}{2} BC, ED = \frac{1}{2} AB$. 由

$$\text{题意可知 } \frac{S_{\text{矩形} FEDB}}{S_{\triangle ABC}} = \frac{EF \cdot DE}{\frac{1}{2} AB \cdot BC} = \frac{\frac{1}{2} BC \cdot \frac{1}{2} AB}{\frac{1}{2} AB \cdot BC} =$$

$\frac{1}{2}$, $\therefore$ 所得矩形的最大面积与原三角形面积的比值为 $\frac{1}{2}$, 故答案为 $\frac{1}{2}$.

(2) 设 $PN = b$. 由题知 $PN \parallel BC$, $\therefore \triangle APN \sim \triangle ABC$. 由题意易知, $PQ = ED = MN$.

$$\therefore AD \perp BC, PN \parallel BC, \therefore AE \perp PN, \therefore \frac{AE}{AD} = \frac{PN}{BC}.$$

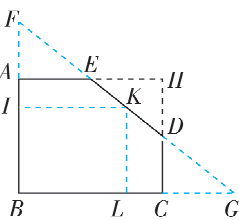
$$\therefore BC = 120, AD = 80, \therefore \frac{80 - PQ}{80} = \frac{b}{120}, \therefore PQ =$$

$$80 - \frac{2b}{3}, \therefore S_{\text{矩形} PQMN} = PN \cdot PQ = b \times \left(80 - \frac{2}{3}b \right) =$$

$$-\frac{2}{3}(b - 60)^2 + 2400,$$

$\therefore$ 当 $b = 60$ 时, $S_{\text{矩形} PQMN}$ 有最大值, 此时 $PN = 60, PQ = 40$, $\therefore$ 矩形 $PQMN$ 的周长为 $2 \times (60 + 40) = 200$.

(3) 如图(1), 延长 BA , DE 交于点 F , 延长 BC , ED 交于点 G , 取 BF 的中点 I , FG 的中点 K , 连接 IK .



图(1)

设 AE 的延长线与 CD

的延长线交于点 H .

由题意知四边形 $ABCH$ 是矩形, $\therefore \angle BAH = \angle H = \angle FAE = 90^\circ, AB = CH, AH = BC$.

$\therefore AB = 32, BC = 40, AE = 20, CD = 16, \therefore EH = AH - AE = BC - AE = 20, DH = CH - CD = AB - CD = 16, \therefore AE = EH, CD = DH$.

在 $\triangle AEF$ 和 $\triangle HED$ 中, $\begin{cases} \angle FAE = \angle DHE, \\ AE = EH, \\ \angle AEF = \angle HED, \end{cases}$

$\therefore \triangle AEF \cong \triangle HED$ (ASA), $\therefore AF = DH = 16$,

$$\therefore BI = \frac{AB + AF}{2} = 24.$$

同理可得 $\triangle CDG \cong \triangle HDE$ (ASA), $\therefore CG = HE = 20$.

过点 K 作 $KL \perp BC$ 于点 L . $\because BI = 24 < 32, \therefore$ 易知 IK, KL 在缺角矩形 $ABCD$ 的内部. 同(1)可知, 此时矩形 $BIKL$ 的面积最大. 由题知 $\angle B = 90^\circ. \therefore KL \perp BC, \therefore \angle KLG = \angle B, \therefore KL \parallel BF, \therefore KL$ 是 $\triangle GBF$ 的中位线, $\therefore BL = \frac{1}{2} BG =$

关键点拨

(4) 延长 BA , CD 交于点 E , 推出 $\triangle BEC$ 为等边三角形是解题的关键.

30, $\therefore$ 矩形 $BIKL$ 的最大面积为 $BL \cdot BI = 30 \times 24 = 720$. 故答案为 720.

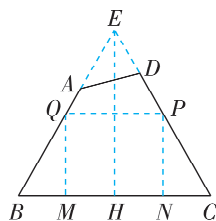
(4) 如图(2), 延长 BA , CD 交于点 E , 过点 E 作 $EH \perp BC$ 于点 H .

$\therefore \angle B = \angle C = 60^\circ,$

$\therefore \angle BEC = 60^\circ,$

$\therefore \triangle BEC$ 为等边三角形,

$\therefore BC = BE = CE$.



图(2)

$$\therefore EH \perp BC, \therefore BH = HC, \angle CEH = \frac{1}{2} \angle BEC = 30^\circ.$$

$$\therefore BC = 108 \text{ cm}, \therefore BH = CH = 54 \text{ cm}, EH = 54\sqrt{3} \text{ cm}, EB = EC = 2BH = 108 \text{ cm}.$$

取 BE, CE 的中点 Q, P , 连接 PQ , 作 $QM \perp BC$ 于 $M, PN \perp BC$ 于 N . $\because AB = 70 \text{ cm}, \therefore AE = 38 \text{ cm}, \therefore BE$ 的中点 Q 在线段 AB 上. $\because CD = 76 \text{ cm}, \therefore DE = 32 \text{ cm}, \therefore CE$ 的中点 P 在线段 CD 上. 由(2)可知, 此时矩形 $PQMN$ 的面积取得最大值, 为 $\frac{1}{4} BC \cdot EH = \frac{1}{4} \times 108 \times 54\sqrt{3} =$

$$1458\sqrt{3} (\text{cm}^2).$$

故答案为 $1458\sqrt{3} \text{ cm}^2$.

2. 【解】(1) 由题意得 $AE = EC, AF = FB$,

$$\therefore EF \parallel BC, EF = \frac{1}{2} BC, \therefore \triangle FEP \sim \triangle CBP,$$

$$\therefore \frac{EF}{BC} = \frac{PF}{PC} = \frac{PE}{PB} = \frac{1}{2}, \therefore PE = \frac{1}{3} BE = 3, PF =$$

$$\frac{1}{3} CF = 2. \therefore BE \perp CF, \therefore \angle EPF = 90^\circ, \therefore EF =$$

$$\sqrt{PE^2 + PF^2} = \sqrt{13}.$$

(2) $\triangle ABC$ 是“中垂三角形”.

证明: 如图, 延长 PD 到 M , 使得 $DM = PD$, 连接 BM, CM .

$\because AD$ 为中线,

$\therefore BD = DC$.

$\because PD = DM, BD = DC, \therefore$ 四边形 $PBMC$ 是平行四边形, $\therefore PB = CM$.

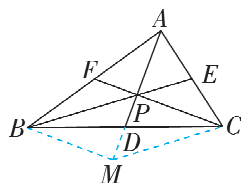
由中线的性质易得 $PB = \frac{2}{3} BE = \frac{20}{3}, PD = DM =$

$$\frac{1}{3} AD = 2, PC = \frac{2}{3} CF = \frac{16}{3}, \therefore PM = 4.$$

$$\therefore PM^2 + PC^2 = 4^2 + \left(\frac{16}{3} \right)^2 = \frac{400}{9}, CM^2 = PB^2 =$$

$$\frac{400}{9}, \therefore CM^2 = PM^2 + PC^2, \therefore \angle CPD = 90^\circ,$$

$\therefore CF \perp AD, \therefore \triangle ABC$ 是“中垂三角形”.



关键点拨

(2) 解题的关键是学会添加适当的辅助线, 构造平行四边形解决问题.

全章综合训练

刷中考

1. **A** 【解析】∵ 四边形 $ABCD$ 是正方形, ∴ $AD \parallel AB$. 由平移的性质得 $EF \parallel AD$, ∴ $GH \parallel AD$. ∵ 点 G 是 OA 的中点, ∴ $OG = AG$, ∴ $OA = 2OG$.
 ∵ $GH \parallel AD$, ∴ $\triangle OGH \sim \triangle OAD$, ∴ $\frac{GH}{AD} = \frac{OG}{OA} = \frac{OG}{2OG} = \frac{1}{2}$, ∴ $\frac{GH}{AB} = \frac{1}{2}$. 故选 A.

2. (1) 【解】∵ $AC = 2\sqrt{5}$, $AE = 2$, $AE \perp CD$,
 ∴ $\angle AEC = 90^\circ$, ∴ $CE = \sqrt{AC^2 - AE^2} = 4$.

(2) 【证明】∵ $\angle BAC = 90^\circ$, $AE \perp CD$,
 ∴ $\angle AED = \angle CAD = 90^\circ$.
 ∵ $\angle ADE = \angle CDA$, ∴ $\triangle ADE \sim \triangle CDA$,
 ∴ $\frac{AD}{CD} = \frac{DE}{AD}$, ∴ $AD^2 = DE \cdot DC$.

∵ $BD = 3AD$, ∴ $BD^2 = 9AD^2 = 9DE \cdot DC$.

(3) 【解】∵ $AE \perp CD$, ∴ $\angle AED = \angle CEA = 90^\circ$,
 $\angle DAE = \angle ACD = 90^\circ - \angle ADC$, ∴ $\triangle AED \sim \triangle CEA$,
 ∴ $\frac{AE}{CE} = \frac{DE}{AE} = \frac{AD}{AC}$, ∴ $\frac{2}{4} = \frac{DE}{2} = \frac{AD}{2\sqrt{5}}$,

∴ $DE = 1$, $AD = \sqrt{5}$, ∴ $BD = 3AD = 3\sqrt{5}$,

∴ $AB = AD + BD = 4\sqrt{5}$, ∴ $AD \cdot AB = \sqrt{5} \times 4\sqrt{5} = 20$.
 ∵ $AC^2 = (2\sqrt{5})^2 = 20$, ∴ $AC^2 = AD \cdot AB$,

∴ $\frac{AD}{AC} = \frac{AC}{AB}$. ∵ $\angle DAC = \angle CAB$, ∴ $\triangle DAC \sim \triangle CAB$,
 ∴ $\angle ACD = \angle ABC$. ∵ $\angle DAE = \angle ACD$,
 ∴ $\angle DAE = \angle ABC$, ∴ $FA = FB$. ∵ $\angle BAC = 90^\circ$,
 ∴ $\angle DAE + \angle FAC = \angle ABC + \angle FCA = 90^\circ$,
 ∴ $\angle FAC = \angle FCA$, ∴ $FC = FA$, ∴ $FA = FB = FC$.

∴ $BC = \sqrt{AC^2 + AB^2} = \sqrt{(2\sqrt{5})^2 + (4\sqrt{5})^2} = 10$,

∴ $AF = \frac{1}{2}BC = 5$, ∴ $EF = AF - AE = 3$.

设 $S_{\triangle ADE} = S$. ∵ $BD = 3AD$, ∴ $S_{\triangle BDE} = 3S$,
 ∴ $S_{\triangle ABE} = S_{\triangle ADE} + S_{\triangle BDE} = 4S$.

∴ $\frac{AE}{EF} = \frac{2}{3}$, ∴ $\frac{S_{\triangle ABE}}{S_{\triangle BEF}} = \frac{AE}{EF} = \frac{2}{3}$, ∴ $S_{\triangle BEF} = 6S$,

∴ $\frac{S_{\triangle BEF}}{S_{\triangle BDE}} = \frac{6S}{3S} = 2$.

3. **B** 【解析】∵ $B'(2, 0)$, $B(4, 0)$, ∴ $OB' = 2$,
 $OB = 4$. ∵ 四边形 $ABCD$ 与四边形 $A'B'C'D'$ 是以原点 O 为位似中心的位似图形, ∴ 易得
 $\frac{A'B'}{AB} = \frac{OB'}{OB}$, 即 $\frac{\sqrt{5}}{AB} = \frac{2}{4}$, ∴ $AB = 2\sqrt{5}$. 故选 B.

关键点拨

本题中由两直线平行可推出 $\triangle OGH \sim \triangle OAD$, 再利用相似三角形对应边成比例可以得出 GH 与 AD 的比值, 最后利用正方形的边长相等即可得出结果.

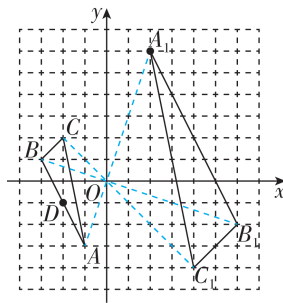
思路分析

(2) 证明 $\triangle ADE \sim \triangle CDA$, 得到 $AD^2 = DE \cdot DC$, 再由 $BD = 3AD$ 代入求证即可.

4. **1:3** 【解析】因为把 $\triangle ABC$ 以原点 O 为位似中心放大得到 $\triangle A'B'C'$, 点 A 和它的对应点 A' 的坐标分别为 $(3, 7)$, $(-9, -21)$, 所以 $\triangle ABC$ 与 $\triangle A'B'C'$ 的相似比为 $3:9 = 7:21 = 1:3$, 故答案为 $1:3$.

5. 【解析】(1) 如图所示, 点 D 即为所求, 点 D 坐标为 $(-2, -1)$.

(2) 如图所示, $\triangle A_1B_1C_1$ 即为所求.



6. **C** 【解析】∵ $AB \perp A'B$, $A'B' \perp A'B$, $PQ \perp A'B$,
 ∴ $AB \parallel PQ \parallel A'B'$, ∴ $\triangle A'PQ \sim \triangle A'AB$,
 $\triangle BPQ \sim \triangle BB'A'$, ∴ $\frac{A'Q}{A'B} = \frac{PQ}{AB} = \frac{6}{10} = \frac{3}{5}$,

∴ $\frac{BQ}{BA'} = \frac{2}{5}$. ∵ $\triangle BPQ \sim \triangle BB'A'$, ∴ $\frac{QP}{A'B'} =$

$\frac{BQ}{BA'} = \frac{2}{5}$, ∴ $A'B' = \frac{5}{2}PQ = 15$ cm. 故选 C.

7. (1) 【证明】∵ $SD \perp AC$, ∴ $\angle SDA = \angle SDC = 90^\circ$,
 ∴ $\angle SDM + \angle MDA = \angle SDB + \angle BDC = 90^\circ$.
 ∵ $\angle MDS = \angle BDS$, ∴ $\angle MDA = \angle BDC$.

(2) 【解】∵ $TE \perp AC$, ∴ $\angle TEA = \angle TEC = 90^\circ$,
 ∴ $\angle TEN + \angle NEA = \angle TEB + \angle BEC = 90^\circ$.

∵ $\angle NET = \angle BET$, ∴ $\angle BEC = \angle NEA$.

由题可知 $BC \perp AC$, $MA \perp AC$, ∴ $\angle BCA = \angle MAC = 90^\circ$,
 ∴ $\triangle BCE \sim \triangle NAE$, ∴ $\frac{BC}{CE} = \frac{NA}{AE}$,

即 $\frac{1.5}{2+2} = \frac{NA}{12-2-2}$, 解得 $NA = 3$.

由(1)知 $\angle MDA = \angle BDC$. ∵ $\angle MAD = \angle BCD = 90^\circ$,
 ∴ $\triangle BCD \sim \triangle MAD$, ∴ $\frac{BC}{CD} = \frac{MA}{AD}$, ∴ $\frac{1.5}{2} =$

$\frac{MA}{12-2}$, 解得 $MA = 7.5$, ∴ $MN = MA - AN = 7.5 - 3 = 4.5$ (米),
 ∴ 此公益广告牌的高度 MN 为 4.5 米.

8. **C** 【解析】设 $AE = x$, $AE + AF = y$. ∵ 四边形 $ABCD$ 是矩形,
 $AB = 3$, E 是 AB 边上的动点, ∴ $BE = 3 - x$, $\angle A = \angle B = 90^\circ$.
 ∵ $EF \perp CE$, ∴ $\angle CEF = 90^\circ$, ∴ $\angle BEC + \angle AEF = 90^\circ = \angle AFE + \angle AEF$,
 ∴ $\angle AFE = \angle BEC$, ∴ $\triangle AFE \sim$

$\triangle BEC$, $\therefore \frac{AF}{BE} = \frac{AE}{BC}$, $\therefore AF = \frac{AE \cdot BE}{BC} = \frac{AE \cdot BE}{\frac{1}{2}AE \cdot BE} = \frac{1}{2}AE \cdot BE$, $\therefore y = AE + AF = AE + \frac{1}{2}AE \cdot BE = x + \frac{1}{2}x(3-x) = -\frac{1}{2}x^2 + \frac{5}{2}x = -\frac{1}{2}\left(x - \frac{5}{2}\right)^2 + \frac{25}{8}$. $\therefore -\frac{1}{2} < 0, 0 < x < 3$, $\therefore$ 当 $x = \frac{5}{2}$ 时, y 有最大值, 最大值是 $\frac{25}{8}$, $\therefore AE + AF$ 的最大值是 $\frac{25}{8}$. 故选 C.

9. $4\sqrt{3}$ 【解析】如图, 过点 A 作 $AD \perp x$ 轴于点 D , 过点 C 作 $CE \perp x$ 轴于点 E , $\therefore CE \parallel AD$. 设等边 $\triangle AOB$ 的边长为 $2a$, $\therefore OA = OB = 2a$, $OD = BD = \frac{1}{2}OB = a$, $\therefore AD = \sqrt{OA^2 - OD^2} = \sqrt{3}a$. $\therefore$ 等边 $\triangle AOB$ 的面积为 $9\sqrt{3}$, $\therefore \frac{1}{2}OB \cdot AD = \frac{1}{2} \times 2a \times \sqrt{3}a = \sqrt{3}a^2 = 9\sqrt{3}$, $\therefore a = 3$ (负值已舍去), $\therefore AD = 3\sqrt{3}$, $OD = 3$. $\therefore AD \parallel CE$, $\therefore \triangle COE \sim \triangle AOD$, $\therefore \frac{AD}{CE} = \frac{OD}{OE} = \frac{OA}{OC}$. $\therefore OA : OC = 3 : 2$, $\therefore \frac{3\sqrt{3}}{CE} = \frac{3}{OE} = \frac{3}{2}$, $\therefore CE = 2\sqrt{3}$, $OE = 2$, $\therefore C(2, 2\sqrt{3})$, $\therefore k = 2 \times 2\sqrt{3} = 4\sqrt{3}$.

10. (1) ①【证明】 $\because CE \parallel AD$, $\therefore \angle DEC = \angle ADM$. $\because AM \perp DE$, $\therefore \angle AMD = 90^\circ$. 由旋转得, $\angle EDC = \angle B = 90^\circ$, $\therefore \angle AMD = \angle EDC = 90^\circ$, $\therefore \triangle ADM \sim \triangle CED$.

②【解】 $\because AD = AE$, $AM \perp DE$, $\therefore DM = \frac{1}{2}DE$.

由①知, $\triangle ADM \sim \triangle CED$, $\therefore \frac{CE}{AD} = \frac{DE}{MD} = 2$.

$\because CE \parallel AD$, $\therefore \triangle AFD \sim \triangle CFE$,

$\therefore \frac{EF}{DF} = \frac{CE}{AD} = 2$, 即 $\frac{EF}{DF}$ 的值为 2.

③【解】 AB 的值为 $\frac{3\sqrt{10}}{4}$.

设 $DE = 6x$. 由②知, $DM = \frac{1}{2}DE = 3x$.

由①知, $\triangle ADM \sim \triangle CED$, $\therefore \frac{AM}{CD} = \frac{DM}{DE} = \frac{1}{2}$,

$\therefore CD = 2AM$. $\because AM \perp DE$,

思路分析

过点 A 作 $AD \perp x$ 轴于点 D , 过点 C 作 $CE \perp x$ 轴于点 E , 设等边 $\triangle AOB$ 的边长为 $2a$, 结合勾股定理和三角形面积公式求出 $AD = 3\sqrt{3}$, $OD = 3$, 证明 $\angle COE \sim \triangle AOD$, 利用相似三角形对应边成比例求出 $CE = 2\sqrt{3}$, $OE = 2$, 从而得到点 C 的坐标, 即可得到 k 的值.

思路分析

(1) ②先推出 $DM = \frac{1}{2}DE$, 进而得出 $\frac{CE}{AD} = \frac{DE}{MD} = 2$, 再证明 $\triangle AFD \sim \triangle CFE$, 进一步求解即可.

$\therefore \angle AMD = 90^\circ = \angle CDE$.

$\therefore \angle AFM = \angle CFD$, $\therefore \triangle AFM \sim \triangle CFD$,

$\therefore \frac{AF}{CF} = \frac{AM}{CD} = \frac{FM}{DF} = \frac{1}{2}$, $\therefore DF = 2FM$, $CF = 2AF$.

$\therefore DF + FM = DM = 3x$, $AF + CF = AC = 3$, $\therefore FM = x$, $DF = 2x$, $AF = 1$, $CF = 2$.

由旋转知, $DE = AB = 6x$, $CE = AC = 3$.

在 $\text{Rt} \triangle CDE$ 中, $CD^2 + DE^2 = CE^2$, $\therefore CD^2 + (6x)^2 = 3^2$, $\therefore CD^2 + 36x^2 = 9$. (I)

在 $\text{Rt} \triangle CDF$ 中, $CD^2 + DF^2 = CF^2$, $\therefore CD^2 + (2x)^2 = 2^2$, $\therefore CD^2 + 4x^2 = 4$. (II)

(I) - (II) 得, $32x^2 = 5$, $\therefore x = \frac{\sqrt{10}}{8}$ (负值舍去), $\therefore AB = 6x = 6 \times \frac{\sqrt{10}}{8} = \frac{3\sqrt{10}}{4}$,

即 AB 的值为 $\frac{3\sqrt{10}}{4}$.

即 AB 的值为 $\frac{3\sqrt{10}}{4}$.

(2) 【解】 $\frac{AB}{AC}$ 的值为 $\frac{2\sqrt{5}}{5}$.

如图, 过点 A 作 $AM \perp DE$

于 M . 设 $AM = a$, $CD = b$.

$\because AD = AE$, $\therefore DM = EM$.

$\because \angle DAE = 90^\circ$, $\therefore AM = \frac{1}{2}DE$, $\therefore DE = 2AM = 2a$,

$EM = a$.

$EM = a$.

延长 DE 至 G , 使 $EG = CD = b$, 连接 AG .

$\because AD = AE$, $\angle DAE = 90^\circ$, $\therefore \angle ADE = \angle AED = 45^\circ$. $\because \angle CDE = 90^\circ$, $\therefore \angle ADC = \angle ADE + \angle CDE = 135^\circ$.

$\because \angle AEG = 180^\circ - \angle AED = 135^\circ$, $\therefore \angle ADC = \angle AEG$, $\therefore \triangle ADC \cong \triangle AEG$ (SAS), $\therefore AG = AC$.

在 $\text{Rt} \triangle AMG$ 中, $AM^2 + MG^2 = AG^2$,

$\therefore a^2 + (a+b)^2 = AC^2$, $\therefore AC^2 = 2a^2 + 2ab + b^2$.

在 $\text{Rt} \triangle ECD$ 中, $CE^2 = DE^2 + CD^2$,

$\therefore CE^2 = (2a)^2 + b^2 = 4a^2 + b^2$.

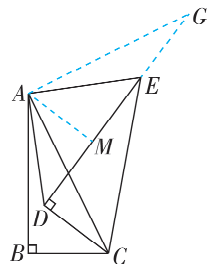
由旋转知, $AC = CE$, $\therefore 2a^2 + 2ab + b^2 = 4a^2 + b^2$,

$\therefore a = b$, $\therefore CD = a$.

由旋转知, $AB = DE = 2a$, $BC = CD = a$.

根据勾股定理得, $AC = \sqrt{AB^2 + BC^2} = \sqrt{5}a$,

$\therefore \frac{AB}{AC} = \frac{2a}{\sqrt{5}a} = \frac{2\sqrt{5}}{5}$, 即 $\frac{AB}{AC}$ 的值为 $\frac{2\sqrt{5}}{5}$.



刷章测

1. B 【解析】 $\because ax = bc, \therefore \frac{a}{c} = \frac{b}{x}$. 观察选项可知, 选项 B 符合题意. 故选 B.

2. C 【解析】 $\because l_1 \parallel l_2 \parallel l_3, \therefore \frac{AB}{BC} = \frac{DE}{EF}$, 即 $\frac{AB}{AC-AB} = \frac{DE}{EF}$. $\because AB = 3, AC = 7, DE = 2.4,$
 $\therefore \frac{3}{7-3} = \frac{2.4}{EF}, \therefore EF = 3.2$. 故选 C.

3. D 【解析】 $\because H$ 是 AB 的黄金分割点, $\therefore AH^2 = BH \cdot AB$. 由题意得, $S_1 = AH^2, S_2 = BH \cdot BC = BH \cdot AB, \therefore S_1 = S_2$, 即 $\frac{S_1}{S_2} = 1$. 故选 D.

4. C 【解析】

①

当 $\angle ACP = \angle B$ 时, $\because \angle A = \angle A, \therefore \triangle ACP \sim \triangle ABC$, ①符合题意

②

当 $\angle APC = \angle ACB$ 时, $\because \angle A = \angle A, \therefore \triangle ACP \sim \triangle ABC$, ②符合题意

③

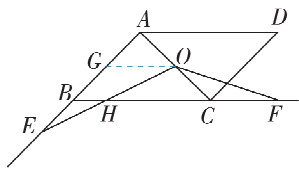
当 $AC^2 = AP \cdot AB$ 时, $\frac{AC}{AP} = \frac{AB}{AC}, \therefore \angle A = \angle A, \therefore \triangle ACP \sim \triangle ABC$, ③符合题意

④

当 $AB \cdot CP = AP \cdot CB$ 时, $\frac{AB}{AP} = \frac{BC}{CP}$, 不能判定 $\triangle APC$ 和 $\triangle ACB$ 相似, ④不符合题意

$\therefore$ 从中随机抽取一个, 能使 $\triangle APC$ 和 $\triangle ACB$ 相似的概率是 0.75. 故选 C.

5. B 【解析】如图, 设 OE, BC 交于点 H , 过点 O 作 $OG \parallel BC$. $\because \angle ABC = 45^\circ, AC = AB = 2, \therefore \angle ACB = \angle ABC = 45^\circ,$



$\therefore \angle BAC = 90^\circ, \angle EBH = \angle OCF = 180^\circ - 45^\circ = 135^\circ, \therefore BC = 2\sqrt{2}. \because OG \parallel BC, \therefore \triangle AOG \sim \triangle ACB, \therefore \frac{AG}{AB} = \frac{OG}{BC} = \frac{AO}{AC}. \because$ 点 O 为 AC 中点, $\therefore OA = OC = \frac{1}{2}AC = 1, \therefore \frac{AG}{AB} = \frac{OG}{BC} = \frac{AO}{AC} = \frac{1}{2}, \therefore AG = \frac{1}{2}AB = 1, OG = \frac{1}{2}BC = \sqrt{2}, \therefore BG = AB - AG = 1, \therefore EG = BE + BG = x + 1. \because OG \parallel BC, \therefore \triangle EBH \sim \triangle EGO, \therefore \frac{BE}{EG} = \frac{BH}{OG},$ 即 $\frac{x}{1+x} = \frac{BH}{\sqrt{2}},$

思路分析

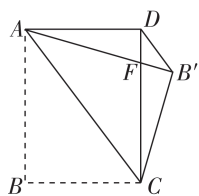
设 AB' 与 CD 交于 F . 先根据平行线的性质得到 $\angle DCA = \angle BAC$, 然后根据折叠的性质得到 $\angle B'AC = \angle BAC$, 从而可得到 $AF = FC$. 根据折叠的性质得到 $\angle FB'C = \angle B = \angle ADC = 90^\circ, CB' = CB = AD$, 证明 $\triangle DAF \cong \triangle B'CF$, 再根据全等三角形的性质得到 $DF = B'F$. 根据勾股定理得到 $AC = \sqrt{AB^2 + BC^2} = 5$. 设 $B'F = DF = x$, 则 $FC = 4 - x$, 根据勾股定理得到 $x = \frac{7}{8}$. 最后根据相似三角形的判定与性质即可得出结论.

思路分析

在 AD 上截取 $AE = 2$, 连接 CE, BE , 可证 $\triangle ACE \sim \triangle ADC$, 进而得出 $BC + \frac{1}{2}CD = BC + CE$, 则 B, C, E 共线时 $BC + \frac{1}{2}CD$ 有最小值, 即为 BE 的长.

$\therefore BH = \frac{\sqrt{2}x}{1+x}, \therefore \angle EBH = 135^\circ, \angle EOF = 135^\circ,$
 $\therefore \angle BEH + \angle BHE = 45^\circ, \angle OFC + \angle OHF = 45^\circ.$
 $\therefore \angle BHE = \angle OHF, \therefore \angle BEH = \angle OFC.$
 又 $\because \angle EBH = \angle OCF, \therefore \triangle EBH \sim \triangle FCO,$
 $\therefore \frac{BE}{CF} = \frac{BH}{OC},$ 即 $\frac{x}{y} = \frac{1+x}{1}, \therefore y = x \cdot \frac{1+x}{\sqrt{2}x} = \frac{\sqrt{2}}{2}(x+1).$ 故选 B.

6. C 【解析】如图, 设 AB' 与 CD 交于点 F . $\because$ 矩形 $ABCD$ 中, $DC \parallel AB,$
 $\therefore \angle DCA = \angle BAC$. 由折叠的性质可知 $\angle B'AC =$



$\angle BAC, \therefore \angle DCA = \angle B'AC, \therefore AF = FC$. 由折叠的性质可知 $\angle FB'C = \angle B = 90^\circ, CB' = CB,$
 $\therefore$ 易得 $AD = CB', \angle FB'C = \angle FDA = 90^\circ$. 在

$\triangle DAF$ 和 $\triangle B'CF$ 中, $\begin{cases} \angle DFA = \angle B'FC, \\ \angle FDA = \angle FB'C, \\ AD = CB', \end{cases}$

$\therefore \triangle DAF \cong \triangle B'CF (AAS), \therefore DF = B'F.$ $\because$ 在矩形 $ABCD$ 中, $AB = 4, BC = 3, \angle B = 90^\circ,$
 $\therefore AC = \sqrt{AB^2 + BC^2} = 5$. 设 $B'F = DF = x$, 则 $FC = 4 - x$. 在 $Rt\triangle B'FC$ 中, 根据勾股定理得 $B'C^2 + B'F^2 = FC^2$, 即 $3^2 + x^2 = (4 - x)^2$, 解得 $x = \frac{7}{8},$

$\therefore FC = \frac{25}{8}. \because AF = CF, DF = B'F, \angle AFC =$

$\angle DFB', \therefore \frac{AF}{B'F} = \frac{CF}{DF}, \therefore \triangle FAC \sim \triangle FB'D,$

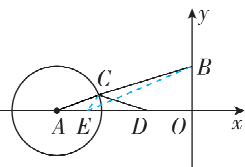
$\therefore \frac{AC}{DB'} = \frac{FC}{DF},$ 即 $\frac{5}{DB'} = \frac{\frac{25}{8}}{\frac{7}{8}},$ 解得 $DB' = 1.4$. 故

选 C.

7. $\frac{3}{5}$ 【解析】 $\because \frac{a}{5} = \frac{b}{8} (a \neq 0), \therefore \frac{b}{a} = \frac{8}{5},$
 $\therefore \frac{b-a}{a} = \frac{b}{a} - 1 = \frac{8}{5} - 1 = \frac{3}{5}.$

8. $2\sqrt{29}$ 【解析】如图, 在 AD 上截取 $AE = 2$, 连接 $CE, BE. \because \frac{AE}{AC} = \frac{AC}{AD} = \frac{1}{2}, \angle CAD$ 为公共角,
 $\therefore \triangle ACE \sim \triangle ADC, \therefore \frac{CE}{CD} = \frac{AC}{AD} = \frac{1}{2}, \therefore CE =$

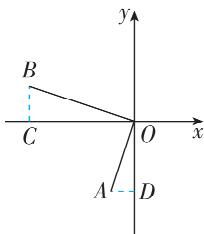
$\frac{1}{2}CD$, $\therefore BC + \frac{1}{2}CD = BC + CE$, $\therefore$ 当 B, C, E 共线时, $BC + \frac{1}{2}CD$ 的值最小, 此时 $BC + \frac{1}{2}CD = BE$. 由题意得, $OB = 4$,



小, 此时 $BC + \frac{1}{2}CD = BE$. 由题意得, $OB = 4$, $OE = 10$, $\therefore BE = \sqrt{OB^2 + OE^2} = 2\sqrt{29}$, $\therefore BC + \frac{1}{2}CD$ 的最小值为 $2\sqrt{29}$, 故答案为 $2\sqrt{29}$.

9. $(-\frac{9}{2}, \frac{3}{2})$ 【解析】如图,

作 $AD \perp y$ 轴于 D , $BC \perp x$ 轴于 C . 根据题意可得 $\angle AOB = 90^\circ$, $\frac{OB}{OA} = \frac{3}{2}$.



$\therefore \angle AOD + \angle AOC = 90^\circ$, $\angle BOC + \angle AOC = 90^\circ$, $\therefore \angle AOD = \angle BOC$. $\therefore \angle ADO = \angle BCO = 90^\circ$, $\therefore \triangle ADO \sim \triangle BCO$, $\therefore \frac{OB}{OA} = \frac{BC}{DA} = \frac{OC}{OD} = \frac{3}{2}$.

$\therefore A(-1, -3)$, $\therefore AD = 1$, $OD = 3$, $\therefore BC = \frac{3}{2}$,

$OC = \frac{9}{2}$, $\therefore B(-\frac{9}{2}, \frac{3}{2})$. 故答案为 $(-\frac{9}{2}, \frac{3}{2})$.

10. $\frac{4\sqrt{2}}{3}$ 【解析】如图, 过

点 D 作 $DH \perp AC$ 于点 H . 设 $AE = x$, 则 $AD = \sqrt{5}x$. $\therefore \angle AHD = \angle ACB =$

90° , $\angle A = \angle A$, $\therefore \triangle ADH \sim \triangle ABC$, $\therefore \frac{DH}{BC} = \frac{AH}{AC}$.

$\therefore BC = 2$, $AC = 4$, $\therefore \frac{DH}{2} = \frac{AH}{4}$, $\therefore \frac{DH}{AH} = \frac{1}{2}$, 即

$AH = 2DH$, $\therefore AD = \sqrt{AH^2 + DH^2} = \sqrt{5}DH$, $\therefore DH = x$, $AH = 2x$, $\therefore EH = x = DH = AE$, $\therefore \triangle DEH$ 为等

腰直角三角形, $\therefore DE = \sqrt{2}x$, $\angle DEH = 45^\circ$, $\therefore \angle AED = 135^\circ$. 由折叠的性质得 $EF = AE =$

DH , $\angle AED = \angle DEF = 135^\circ$, $DF = AD = \sqrt{5}x$, $\therefore \angle FEG = \angle DEF - \angle DEH = 90^\circ$, $\therefore \angle FEG =$

$\angle DHG$. $\therefore \angle EGF = \angle DGH$, $EF = DH$, $\therefore \triangle EFG \cong \triangle HDG$ (AAS), $\therefore DG = FG$,

$\therefore DG = \frac{\sqrt{5}}{2}x$. $\therefore AB = \sqrt{AC^2 + BC^2} = 2\sqrt{5}$,

$\therefore BD = AB - AD = 2\sqrt{5} - \sqrt{5}x$. $\therefore DB = DG$,

$\therefore 2\sqrt{5} - \sqrt{5}x = \frac{\sqrt{5}}{2}x$, $\therefore x = \frac{4}{3}$, $\therefore DE = \sqrt{2}x =$

关键点拨

作 $AD \perp y$ 轴于 D , $BC \perp x$ 轴于 C , 利用“两角分别相等的两个三角形相似”推出 $\triangle ADO \sim \triangle BCO$ 是解题的关键.

关键点拨

过点 D 作 $DH \perp AC$ 于点 H , 推出 $\triangle ADH \sim \triangle ABC$, 进而得到 $\triangle DEH$ 为等腰直角三角形是解题的关键.

思路分析

过 E 作 $EG \perp BC$ 于 G . 依据 $\triangle ABC \sim \triangle ADE$, 即可得出 $\frac{AC}{AE} =$

$\frac{4}{7}$, 进而得出

$\frac{AC}{EC} = \frac{4}{3}$, 依据

$\triangle ACF \sim \triangle ECG$,

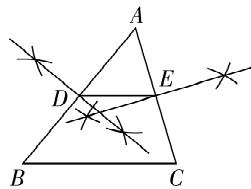
即可得到 $\frac{AF}{EG} =$

$\frac{AC}{EC}$, 进而得出

AF 的长.

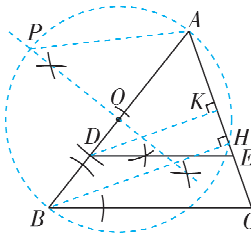
$\frac{4\sqrt{2}}{3}$. 故答案为 $\frac{4\sqrt{2}}{3}$.

11. 【解】(1) 如图(1), 分别作 AB 和 AC 的垂直平分线, 垂足分别为点 D, E , 连接 DE , 点 D, E 即为所求作. (作法不唯一)



图(1)

(2) 如图(2), 先作 AB 的垂直平分线得到 AB 的中点 O , 再以 AB 为直径作 $\odot O$, AB 的垂直平分线交 $\odot O$ 于 P 点, 接着在 AB 截取 $AD = AP$, 然后作 $\angle ADE = \angle ABC$ 交 AC 于 E 点, $\triangle ADE$ 即为所作. (作法不唯一)



图(2)

由作图知, $AD = AP = \sqrt{2}OA = \frac{\sqrt{2}}{2}AB$, $DE \parallel BC$,

$\therefore \frac{AE}{AC} = \frac{AD}{AB}$, $\therefore AE = \frac{\sqrt{2}}{2}AC$,

作 $BH \perp AC$ 于点 H , 作 $DK \perp AC$ 于点 K ,

则 $DK \parallel BH$, $\therefore \triangle ADK \sim \triangle ABH$, $\therefore \frac{DK}{BH} = \frac{AD}{AB}$,

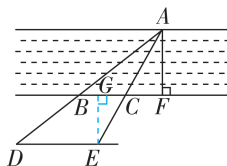
$\therefore DK = \frac{\sqrt{2}}{2}BH$.

$\therefore S_{\triangle ABC} = \frac{1}{2}AC \cdot BH$, $S_{\triangle ADE} = \frac{1}{2}AE \cdot DK$,

$\therefore \frac{S_{\triangle ADE}}{S_{\triangle ABC}} = \frac{\frac{1}{2}AE \cdot DK}{\frac{1}{2}AC \cdot BH} = \frac{\frac{\sqrt{2}}{2}AC \cdot \frac{\sqrt{2}}{2}BH}{AC \cdot BH} = \frac{1}{2}$,

即 $\triangle ADE$ 的面积等于 $\triangle ABC$ 的面积的一半.

12. 【解】如图所示, 过 E 作 $EG \perp BC$ 于 G .



$\therefore DE \parallel BC$, $\therefore \triangle ABC \sim \triangle ADE$, $\therefore \frac{AC}{AE} = \frac{BC}{DE} =$